Problem sheet for AI Principles, semester 2, week 2

**Question 1.** Describe, in your own words, the difference between inductive and deductive reasoning. You may want to use examples.

**Question 2.** Are all thoughts a form of reasoning? Explain your answer.

**Question 3.** What is the difference between an invalid argument and a false statement?

**Question 4.** Indicate whether each of the following sentences is True or False.

(i) Every premise of a valid argument is true [F]
(ii) Every invalid argument has a false conclusion [F]
(iii) Every valid argument has exactly two premises [F]
(iv) Some valid arguments have false conclusions [T]
(v) Some valid arguments have a false conclusion despite having premises that are all true [F]
(vi) A sound argument cannot have a false conclusion [T]
(vii) Some sound arguments are invalid [F]
(viii) Some unsound arguments have true premises [T]
(ix) Premises of sound arguments entail their conclusions [T]
(x) If an argument has true premises and a true conclusion then it is sound [F]

**Question 5.** What are the limitations of doing reasoning with natural language?

**Question 6.** What are the elements of formal systems found in the mu puzzle? What elements of a formal system are missing in the mu puzzle? What is a proof in the mu puzzle?

**Question 7.** Indicate whether each of the following sentences is True or False.

(i) Formal systems must possess semantics [F]
(ii) Semantics involve mapping sentences in a logic to aspects of the real world [T]
(iii) Sentences in logics are never given any meaning [F]
(iv) Different interpretations can be given to sentences in logics, that give rise to different meanings [T]
(v) When a logical sentence A, is true because it logically follows from another sentence, B, we say that A entails B [T]
(vi) If we know that A entails B, and an inference rules shows that ¬ B can be proved from A, we say that the inference rule in question is sound.[F]
(7) Every logic possesses complete inference rules [F]

**Question 8.** List the symbols and their natural language meanings for the five propositional connectives

**Question 9.** What are the rules for forming wff in Propositional logic

**Question 10.** Which of the following expressions are not wffs in Propositional logic?

(a) P [wff]
(b) (P [not wff]
(c) (P) [not wff]
(d) (A → B) [wff]
(e) (A → B) ↔ (P → Q) [wff]
(f) −(P)[not wff]
(g) (A → B) → R [wff]
(h) −(A → B) [wff]
(i) −(P)→ R [not wff]
(j) A ∨ B [wff]
(k) A ∧ B [wff]
(l) −(A ∨ B) [wff]
(m) −(A ∧ B) [not wff]
(n) (−(P ∧ Q) ∧ (D ↔ (Z ∨ K))) [wff]
(o) (−(P ∧ Q) ∧ (DD ↔ (ZK ∨ KZ))) [not wff]
(p) A ∨ → B [not wff]

**Question 11.** List the order of precedence for the logical connectives

**Question 12.** Is the following formulae ambiguous?:

G → −P ∧ D ↔ Z ∨ K

**Question 13.** Change one or more connectives to make the formulae in the previous question ambiguous.

**Question 14.** Use this translation scheme to formally represent (symbolise) the following English sentences into Propositional Logic

P: A purpose of punishment is deterrence
Q: Capital punishment is an effective deterrent
R: Capital punishment should be continued
S: Capital punishment is used in the United States
T: A purpose of punishment is retribution

(i) If retribution is a purpose of punishment but deterrence is not, then capital punishment should not be continued
\((T \land \neg P) \rightarrow \neg R\)

(ii) Capital punishment should be continued even though capital punishment is not an effective deterrent provided that a purpose of punishment is retribution in addition to deterrence
\((T \land P) \rightarrow (R \land \neg Q)\)

**Question 15.** Use this translation scheme to formally represent (symbolise) the following English sentences into Propositional Logic

P: John dances
Q: Mary dances
R: Bill dances
S: John is happy
T: Mary is happy
U: Bill is happy

(1) John is dancing but Mary is not dancing
\(P \land \neg Q\)

(2) If John does not dance, then Mary will not be happy
\(\neg P \rightarrow \neg T\)

(3) John’s dancing is sufficient to make Mary happy
\(P \rightarrow T\)

(4) John’s dancing is necessary to make Mary happy
\(T \rightarrow P\)

(5) John will not dance unless Mary is happy
\(\neg P \lor T\)

or
\(\neg T \rightarrow \neg P\)

(6) If John’s dancing is necessary for Mary to be happy, Bill will be unhappy
\((T \rightarrow P) \rightarrow \neg U\)

(7) If Mary dances although John is not happy, Bill will dance
\((Q \land \neg S) \rightarrow R\)
(8) If neither John nor Bill is dancing, Mary is not happy
\neg (P \lor R) \rightarrow \neg T

(9) Mary is not happy unless either John or Bill is dancing
\neg T \lor (P \lor R)
or
\neg (P \lor R) \rightarrow \neg T

(10) Mary will be happy if both John and Bill dance
(P \land R) \rightarrow T

(11) Although neither John nor Bill is dancing, Mary is happy
T \land \neg (P \land R)

(12) If Bill dances, then if Mary dances John will too
R \rightarrow (Q \rightarrow P)

(13) Mary will be happy only if Bill is happy
T \rightarrow U

(14) Neither John nor Bill will dance if Mary is not happy
\neg T \rightarrow \neg (P \lor R)

(15) If Mary dances only if Bill dances and John dances only if Mary dances, then John dances only if Bill dances
(Q \rightarrow R) \land (P \rightarrow Q) \rightarrow (P \rightarrow R)

(16) Mary will dance if John or Bill but not both dance
(P \lor R) \land \neg (P \land R) \rightarrow Q

(17) If John dances and so does Mary, but Bill does not, then Mary will not be happy but John and Bill will
(P \land Q) \land \neg R \rightarrow (\neg T \land (S \land U))

(18) Mary will be happy if and only if John is happy
T \leftrightarrow S

(19) Provided that Bill is unhappy, John will not dance unless Mary is dancing
\neg U \rightarrow (\neg P \lor Q)
or
(\neg U \rightarrow (\neg Q \rightarrow \neg P)
(20) If John dances on the condition that if he dances Mary dances, then he dances

\[(P \rightarrow Q) \rightarrow P\]

**Question 16.** Produce truth tables for each of the five propositional connectives, comment on any of the results that surprises you.

**Question 17.** Read the top of page 24 in Callan, and then calculate how many rows a truth table will need to evaluate an expression which possess five different atomic propositions

**Question 18.** Use Truth tables to evaluate the following expressions

1. \[P \lor (\neg P \lor Q)\]

\[
\begin{array}{c|c|c|c|c}
P & Q & \neg P & P \lor (\neg P \lor Q) \\
T & T & F & T \\
T & F & T & T \\
F & T & F & T \\
F & F & T & T \\
\end{array}
\]

2. \[\neg (P \land Q) \lor P\]

\[
\begin{array}{c|c|c|c|c}
P & Q & P \land Q & \neg (P \land Q) & \neg (P \land Q) \lor P \\
T & T & T & F & T \\
T & F & F & T & T \\
F & T & F & T & T \\
F & F & F & T & T \\
\end{array}
\]

3. \[\neg (P \rightarrow Q) \rightarrow P\]

\[
\begin{array}{c|c|c|c|c}
P & Q & P \rightarrow Q & \neg (P \rightarrow Q) & \neg (P \rightarrow Q) \rightarrow P \\
T & T & T & F & T \\
T & F & F & T & T \\
F & T & F & T & T \\
F & F & T & T & T \\
\end{array}
\]

4. \[(P \lor Q) \lor (\neg P \land Q)\]

\[
\begin{array}{c|c|c|c|c|c|c}
P & Q & (P \lor Q) & (\neg P \land Q) & (P \lor Q) \lor (\neg P \land Q) \\
T & T & T & F & T \\
T & F & T & F & T \\
F & T & T & F & T \\
F & F & T & F & F \\
\end{array}
\]

5. \[P \lor Q \rightarrow R \lor \neg P\]

\[
\begin{array}{c|c|c|c|c|c|c|c}
P & Q & R & (P \lor Q) & (R \lor \neg P) & (P \lor Q) \lor (R \lor \neg P) & P \lor Q \rightarrow R \lor \neg P \\
T & T & T & T & T & T & T \\
T & F & T & T & F & T & F \\
F & T & T & T & T & T & T \\
F & F & T & T & T & T & T \\
\end{array}
\]

6. \[R \leftrightarrow \neg P \lor (R \land Q)\]
Question 19. Arguments and validity

(i) Is the following argument valid? (HINT: symbolise and use a truth table to see if the argument is a tautology)

1 John’s keys are in the car or hung up in the office
2 John’s keys are not in the car
3 Therefore, John’s keys are hung up in the office
(from Callan, page 25-26)

**Question 20.** Explain why, for large problems, the manual use of truth tables to prove logical conclusions is cumbersome? Explain why this may not be a problem for automated reasoning?