Problem sheet for AI Principles, semester 2, week 3

**Question 1.** Which of the following statements is false:
- The validity of inference rules can be tested using Truth Tables
- A tautology is a necessarily true statement
- If an inference rule is invalid it can still be sound
- All logics have a set of inference rules that are sound and complete
- The symbol for entailment is $\models$
- The symbol for inference is $\vdash$
- Entailment and inference mean the same thing
- There are only six inference rules for Propositional Logic

**Question 2.** List the six inference rules for Propositional Logic that were given in your notes to lecture six. Use the internet or textbooks to find details of other inference rules for Propositional Logic.

**Question 3.** Using rules of deduction prove the following expressions from each set of assumptions. Use the format from the lectures to present your proofs (that is, number each new expression formed during the proof, and for each new expression add the abbreviation for the inference rule that was used)

(i) When the assumptions given are:
1 $\neg\neg T$
Prove $T$

(ii) When assumptions given are:
1 $A$
Prove $A \lor B$

(iii) When assumptions given are:
1 $M \rightarrow O$
2 $N \rightarrow O$
(iv) When assumptions given are:
1 \( E \land F \)
Prove \( F \)

(v) When assumptions given are:
1 \( G \)
2 \( H \)
Prove \( G \land H \)

(vi) When assumptions given are:
1 \( G \land H \)
Prove \( G \)

(vii) When assumptions given are:
1 \( G \rightarrow I \)
2 \( G \lor J \)
3 \( J \rightarrow I \)
Prove \( I \)

(viii) When assumptions given are:
1 \( (K \rightarrow L) \land (M \lor N) \)
2 \( M \rightarrow O \)
3 N → O
4 O → K
Prove L

(ix) When assumptions given are:
1 G → I
2 G ∨ J
3 J → I
Prove I

(x) When the assumptions given are:
1 (p ∧ q) → q
2 p
3 s
4 q → r
Prove r

Question 4. Formally represent (symbolise) the following English sentences into Predicate Logic
(i) Not all cars have four wheels
(ii) Some men are footballers, but not all footballers are men
(iii) Among spiders, only black widows and tarantulas are deadly
(iv) No romantic buys flowers unless he has a date
(v) No animal that tunnels underground can fly
(vi) All dogs are not cats
(vii) Not all Policemen uphold the law
(viii) A mammal with wings is a bat
(ix) Something lost something
(x) Something lost everything
(xi) Everything lost something
(xii) Everything lost everything
(xiii) Everything missed itself
(xiv) Something missed itself

**Question 5.** What are the rules for forming wff in First Order Predicate Logic? (find these in Callan)

**Question 6.** List all the logical equivalences for Propositional Logic from Callan pages 24-26 (you will use these in the next question)

**Question 7.** Convert the following sentences in Propositional Logic into conjunctive normal form

(i) convert the wff \( P \lor (Q \land R) \)

(ii) convert the wff \( P \lor \neg(Q \land \neg R) \)

(iii) convert the wff \( P \lor \neg(\neg Q \land R) \)

(iv) convert the wff \( \neg P \rightarrow (Q \land R) \)

(v) convert the wff \( \neg(P \rightarrow Q) \lor (R \rightarrow P) \)

(vi) convert the wff \( (\neg P \land (\neg Q \rightarrow R)) \rightarrow S \)