An Efficient Jacobian Reduction Method For Image Reconstruction Using Diffuse Optical Tomography

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Abstract: Image reconstruction using Diffuse Optical Tomography requires an accurate inverse model which is usually computationally expensive. We present a Jacobian reduction method which uses the magnitude of the sensitivity to reduce the Jacobian and reconstruct images without detriment to the final solution.

OCIS codes: (100.3190) Inverse problems; (170.3660) Light propagation in tissues; (170.4580)

1. Introduction

Near-Infrared (NIR) Diffuse Optical Tomography (DOT) is a non-invasive imaging technique using wavelengths between 650nm and 900nm. Optical fibers at the surface of the volume inject light into the surrounding tissue and emergent light is measured at the boundary. The measured data is then used in a light propagation model to determine the optical properties of the tissue [1]. Diffuse optical imaging has become an important tool in medical imaging research for brain function activity as well as breast cancer detection and characterization [2].

Much work has gone into the production of accurate forward models to describe the path of the photons within the tissue and the inverse model to efficiently recover images of the optical parameters [2]. For the inverse problem several image reconstruction techniques have been developed. The use of full Newtonian optimization schemes have been used widely where a Jacobian sensitivity matrix, which relates the boundary data to a change in optical parameters, is calculated and the variant is inverted [3]. Alternatively, gradient based minimization schemes have been used which do not require a calculation of a Jacobian matrix [4]. Although these techniques are computationally more efficient, they are difficult to implement and require a non-straightforward optimization.

The problem of DOT image reconstruction is therefore two-fold; the model to describe the photon distribution within the tissue must be accurate and the inverse problem must be reliable and computationally efficient at estimating the optical properties within the tissue. Numerical algorithms based on the Finite Element Method (FEM) for the forward model rely on the accurate definition of the volume and the mesh discretization must be adequate for the calculation. It has been found that for an accurate forward model, the FEM mesh resolution must be of a high quality [5]. On the other hand, the inverse problem is ill-posed and usually underdetermined since there is typically far more unknowns than boundary measurements. Although, strategies such as using a second mesh basis for the inverse problem have tackled these issues.

In this work we present a method which reduces the size of the Jacobian matrix without compromising the numerical accuracy of the both the forward and inverse models. A fine mesh is used for the calculation of the forward model however nodes within the mesh which have a small sensitivity to the measured data are removed efficiently to reduce the size of the matrix. This process, with a tolerance of 1% and for a simple geometry, reduces size of the Jacobian matrix by 8% and the computation time by 6% and the memory by 15% and has the capacity to make much larger gains for more complicated problems.

2. Theory

Since transport of NIR light in tissue is dominated by scattering, the fluence can be accurately described by the diffusion equation given by

$$-\nabla\cdot\kappa(\mathbf{r})\nabla\Phi(\mathbf{r},\omega)+\left(\mu_a(\mathbf{r})+\frac{i\omega}{c_m(\mathbf{r})}\right)\Phi(\mathbf{r},\omega)=q_0(\mathbf{r},\omega), \qquad (1)$$

where $q_0(\mathbf{r},\omega)$ is an isotropic source, $\Phi(\mathbf{r},\omega)$ is the photon fluence rate at position r and modulation frequency ω. κ is the diffusion coefficient given by

$$\kappa = 1/3(\mu_a + \mu_s') \ . \qquad (2)$$

μ_a and μ_s' are absorption and reduced scattering coefficients respectively and $c_m(r)$ is the speed of light in the medium.

The inverse problem has the aim of finding the optical properties, $\mu = [\mu_a, \mu_s']$, at each node of the FEM mesh representing the tissue by finding the minimum between the measured data and the calculated data using a modified Levenberg-Marquardt minimization approach. Here we repeatedly solve

$$\left(J^T J + \lambda I\right)^{-1} J^T \delta\Phi = \delta\mu \ . \qquad (3)$$

$\delta\mu$ is the update vector for the optical properties and J is the Jacobian matrix. λ is a regularization factor which is scaled by the maximum of the diagonal of J^TJ and reduced at each iteration.

3. The Jacobian Reduction Method

The Jacobian matrix relates the change in boundary data due to a change in the optical parameters and is of the size number of measurements, *nm*, by number of nodes or unknowns, *nn*. If for simplicity we consider the change in log amplitude with absorption, the Jacobian takes the form

$$J = \begin{bmatrix} \frac{\delta \ln I_1}{\delta\mu_{a1}} & \frac{\delta \ln I_1}{\delta\mu_{a2}} & \cdots & \frac{\delta \ln I_1}{\delta\mu_{ann}} \\ \frac{\delta \ln I_2}{\delta\mu_{a1}} & \frac{\delta \ln I_2}{\delta\mu_{a2}} & \cdots & \frac{\delta \ln I_2}{\delta\mu_{ann}} \\ \frac{\delta \ln I_3}{\delta\mu_{a1}} & \frac{\delta \ln I_3}{\delta\mu_{a2}} & \cdots & \frac{\delta \ln I_3}{\delta\mu_{ann}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\delta \ln I_{nm}}{\delta\mu_{a1}} & \frac{\delta \ln I_{nm}}{\delta\mu_{a2}} & \cdots & \frac{\delta \ln I_{nm}}{\delta\mu_{ann}} \end{bmatrix} . \qquad (4)$$

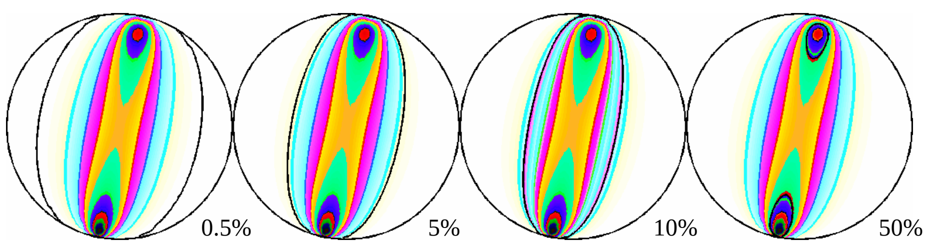

Figure 1. Normalised sensitivity for a circular model with a single source and detector. The contours show the level of sensitivity.

For the inverse problem we require the inverse of the Hessian J^TJ which has the order of *nn* by *nn*. This requires nn^3 operations and nn^2 memory [6]. The greatest sensitivity is always found near to the source-detector plane as shown by figure 1. Therefore the sensitivity far from the source is likely to be small. These regions where the sensitivity is lower than a given threshold, typically 1%, can be removed as they contribute little to the update. Thus the process reduces the size of the Hessian, the memory requirements and the computation time.

4. Experimental Phantom Results

A multilayer gelatin cylindrical phantom of radius 43mm and height 40mm was fabricated with differing optical properties using different concentrations of India ink for absorption and TiO_2 for scattering. Different layers of gelatin were constructed by successively hardening gel solutions containing different amounts of ink and TiO_2. A cylindrical hole of radius 8mm was placed approximately 20mm from the centre with optical properties μ_a = 0.02 mm^{-1} and μ_s' = 1.2 mm^{-1} to replicate a tumor. The fibroglandular layer has optical properties of 0.01 mm^{-1} and 1.0 mm^{-1} for absorption and scattering respectively and has a radius of 38mm. The outer layer of thickness 5mm, similar to a typical fatty breast layer, has the optical properties 0.0065 mm^{-1} and 0.65 mm^{-1} (at 785 *nm*) for absorption and reduced scatter respectively. NIR Data was collected at a modulation frequency of 100 *MHz* using 16 optical fibers placed at the mid-height of the phantom giving rise to 480 data points.

Images were reconstructed for absorption and scattering using both log amplitude and phase using a mesh of 8990 nodes corresponding to a 44803 tetrahedral elements. The initial guess for the optical parameters was assumed to be homogeneous and that of the values of the outer layer. The reconstruction was carried out on a pixel basis of $20 \times 20 \times 20$ using a threshold of 0%, 1%, 5% and 10% for the reduction scheme as shown by figure 2. Here a 0% threshold refers to the normal reconstruction method, without the reduction of the Jacobian.

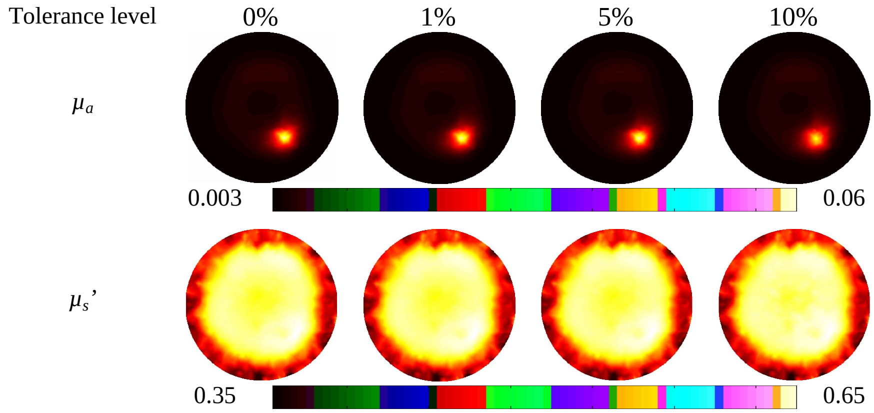

Figure 2. The coronal cross-section of the reconstructed images for μ_a and μ_s' of the cylindrical phantom using the reduced and non-reduced (0% threshold) Jacobian.

For each reconstruction method there is a large contrast in the absorption but a low contrast in the scattering. There is no qualitative and quantitative difference between the reconstruction using a tolerance of 1% and the standard reconstruction (0%). As the tolerance increases it is possible to achieve large saving in memory but the images have more artifacts and quantitative accuracy decreases. At a tolerance of 1% the number of nodes used within the reconstruction is reduced by 8% and saving 0.15 Gb of memory and reducing the computation time by 6%. However, with a tolerance of 10%, the number of nodes reduces by 51% saving 0.73Gb and reducing the computation time to 48% of the normal reconstruction technique.

5. Discussion

In this work we have shown that an efficient Jacobian reduction technique can reduce the computation time and memory requirement of the inversion of the Hessian matrix in the inverse problem without impinging on the qualitative and quantitative accuracy of the reconstructed images. This is important for the reconstruction of images especially when using a fine mesh of a large complicated volume with many unknowns which would usually be computationally inefficient.

6. Acknowledgements

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7. References

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