

Big-step operational semantics overview sheet

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Lambda calculus: beta reduction

Beta reduction \rightarrow_β :

$$\overline{((\lambda x.M_1) M_2) \rightarrow_\beta (M_1[x \mapsto M_2])}$$

Compatible closure of \rightarrow_β :

$$\frac{M_1 \rightarrow_\beta M_2}{(M_1 M_3) \rightarrow_\beta (M_2 M_3)} \quad \frac{M_1 \rightarrow_\beta M_2}{(M_3 M_1) \rightarrow_\beta (M_3 M_2)} \quad \frac{M_1 \rightarrow_\beta M_2}{(\lambda x.M_1) \rightarrow_\beta (\lambda x.M_2)}$$

Reflexive-transitive closure \rightarrow_β^* of \rightarrow_β :

$$\frac{}{M \rightarrow_\beta^* M} \quad \frac{M_1 \rightarrow_\beta^* M_2 \quad M_2 \rightarrow_\beta^* M_3}{M_1 \rightarrow_\beta^* M_3}$$

Values

$$\begin{array}{l} V ::= n \\ \quad | \quad x \\ \quad | \quad \lambda x.M \end{array}$$

Big-step evaluation for call-by-name

$$\frac{}{V \Downarrow V} \quad \frac{M_1 \Downarrow (\lambda x.M_3) \quad M_3[x \mapsto M_2] \Downarrow V}{(M_1 M_2) \Downarrow V}$$

Big-step evaluation for call-by-value

$$\frac{}{V \Downarrow V} \quad \frac{M_1 \Downarrow (\lambda x.M_3) \quad M_2 \Downarrow V_1 \quad M_3[x \mapsto V_1] \Downarrow V_2}{(M_1 M_2) \Downarrow V_2}$$