

## Outline solutions for the class test

### Question 1.

$$\frac{\frac{\lambda f.(f f) \Downarrow \lambda f.(f f)}{\lambda f.(f f) \Downarrow \lambda f.(f f)} \quad \frac{\lambda z.z \Downarrow \lambda z.z}{\lambda z.z \Downarrow \lambda z.z} \quad \frac{\frac{\lambda z.z \Downarrow \lambda z.z \quad \lambda z.z \Downarrow \lambda z.z \quad z[z \mapsto \lambda z.z] \Downarrow \lambda z.z}{(f f)[f \mapsto \lambda z.z] \Downarrow \lambda z.z}}{(\lambda f.(f f))(\lambda z.z) \Downarrow \lambda z.z}}$$

**Question 2.** We can only reuse the same term in a purely functional language. If there are effects, such as assignment, two occurrences that are syntactically identical may evaluate to different values. In the semantics, that is due to the fact that an evaluation judgement does not solely depend on the term, but also on the state, which may have been changed. Hence the initial state of the second occurrence may be different, leading to a different value to be produced.

More formally, we can have a state change in the evaluation of the first occurrence:

$$s_1 \vdash (M 1) \Downarrow V_1, s_2$$

The second occurrence of the same term is then evaluated in the new state rather than the initial  $s_1$  and hence it may produce a different result.

In terms of different effects, it is also worth pointing out that  $(M 1)$  needs to do both a read and a write of some state.

### Question 3.

- In the pure  $\lambda$ -calculus, the rewrite step is correct, as it is a beta reduction. Due to Church-Rosser, we can do no harm in performing arbitrary beta reductions.
- In a language with state, the optimization is not correct. If we delete the call to  $g$  entirely, we also omit any effects on the state it may have, changing the meaning of the program.
- In a language with control operations, the situation is similar to the one for state. Deleting a function call omits any effect. The original term may jump if the function in the argument does, but the rewritten term deletes these possible effects, which does not preserve meaning.