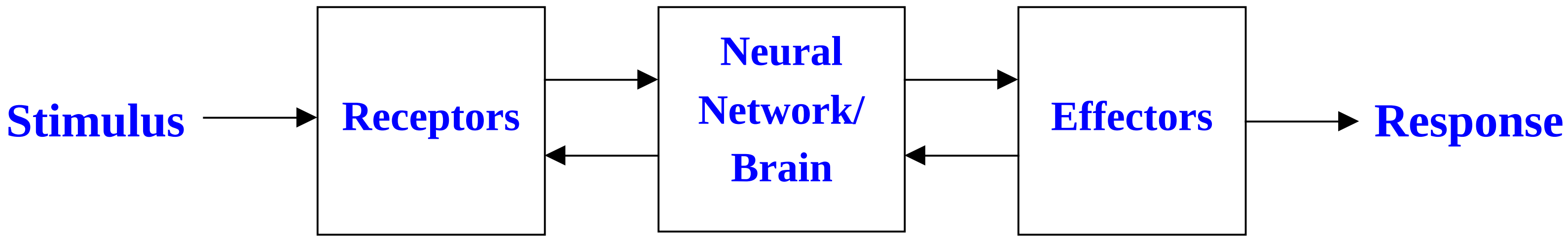
Biological Neurons and Neural Networks, Artificial Neurons

Introduction to Neural Networks : Lecture 2

1. Organization of the Nervous System and Brain
2. Brains versus Computers: Some Numbers
3. Biological Neurons and Neural Networks
4. The McCulloch-Pitts Neuron
5. Some Useful Notation: Vectors, Matrices, Functions
6. The McCulloch-Pitts Neuron Equation

The Nervous System

The human nervous system can be broken down into three stages that may be represented in block diagram form as:



The receptors collect information from the environment – e.g. photons on the retina.

The effectors generate interactions with the environment – e.g. activate muscles.

The flow of information/activation is represented by arrows – feedforward and feedback.

Naturally, in this module we will be primarily concerned with the neural network in the middle.

Levels of Brain Organization

The brain contains both large scale and small scale anatomical structures and different functions take place at higher and lower levels.

There is a hierarchy of interwoven levels of organization:

1. Molecules and Ions
2. Synapses
3. Neuronal microcircuits
4. Dendritic trees
5. **Neurons**
6. **Local circuits**
7. Inter-regional circuits
8. Central nervous system

The ANNs we study in this module are crude approximations to levels 5 and 6.

Brains versus Computers : Some numbers

1. There are approximately 10 billion neurons in the human cortex, compared with 10 of thousands of processors in the most powerful parallel computers.

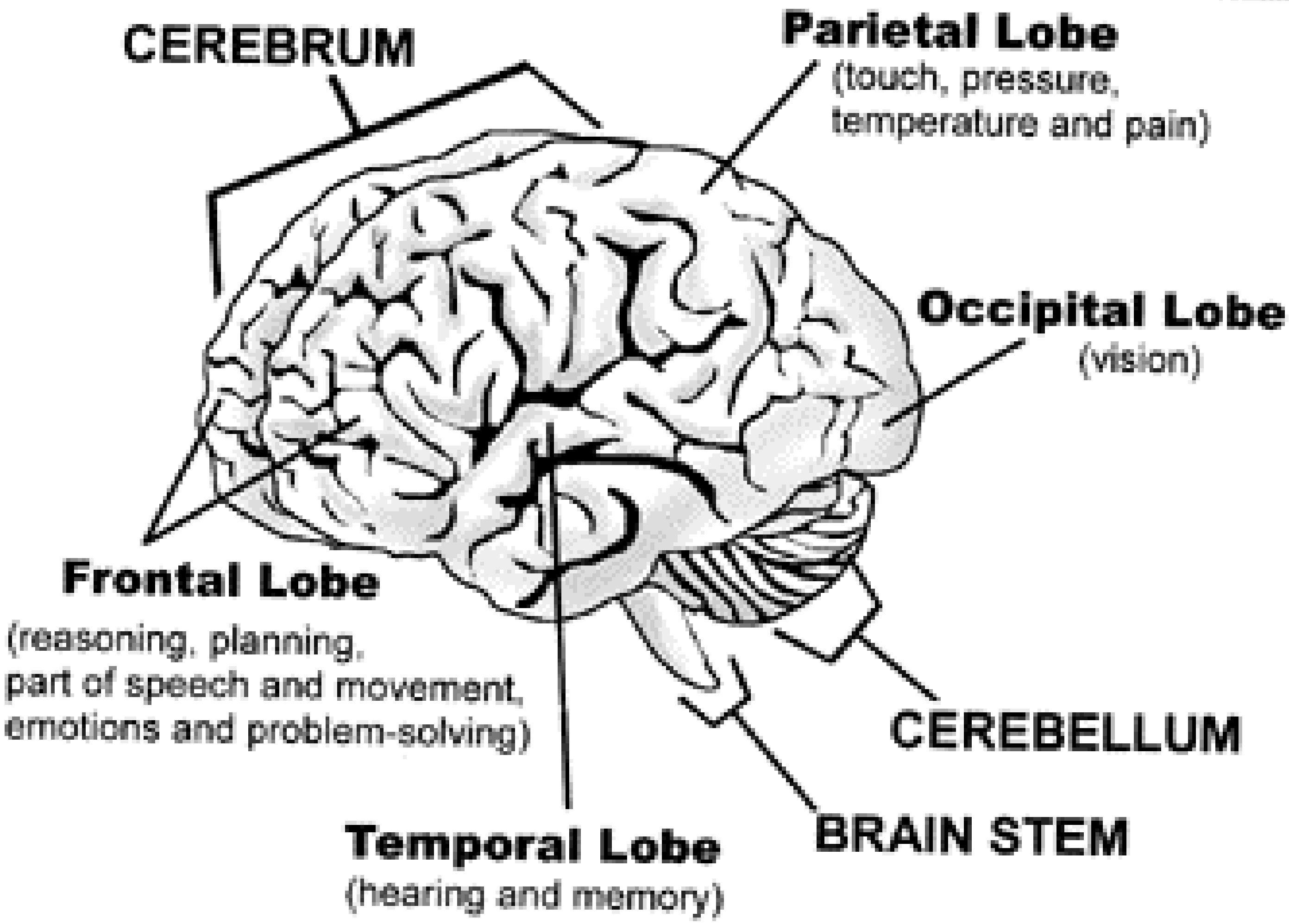
2. Each biological neuron is connected to several thousands of other neurons, similar to the connectivity in powerful parallel computers.

3. Lack of processing units can be compensated by speed. The typical operating speeds of biological neurons is measured in milliseconds (10^{-3} s), while a silicon chip can operate in nanoseconds (10^{-9} s).

4. The human brain is extremely energy efficient, using approximately 10^{-16} joules per operation per second, whereas the best computers today use around 10^{-6} joules per operation per second.

5. Brains have been evolving for tens of millions of years, computers have been evolving for tens of decades.

Structure of a Human Brain



Slice Through a Real Brain

http://medlib.med.utah.edu/WebPath/HISTHTML/NEURANAT/NEURANCA.html

Basic Components of Biological Neurons

1. The majority of ***neurons*** encode their activations or outputs as a series of brief electrical pulses (i.e. spikes or action potentials).

2. The neuron's ***cell body (soma)*** processes the incoming activations and converts them into output activations.

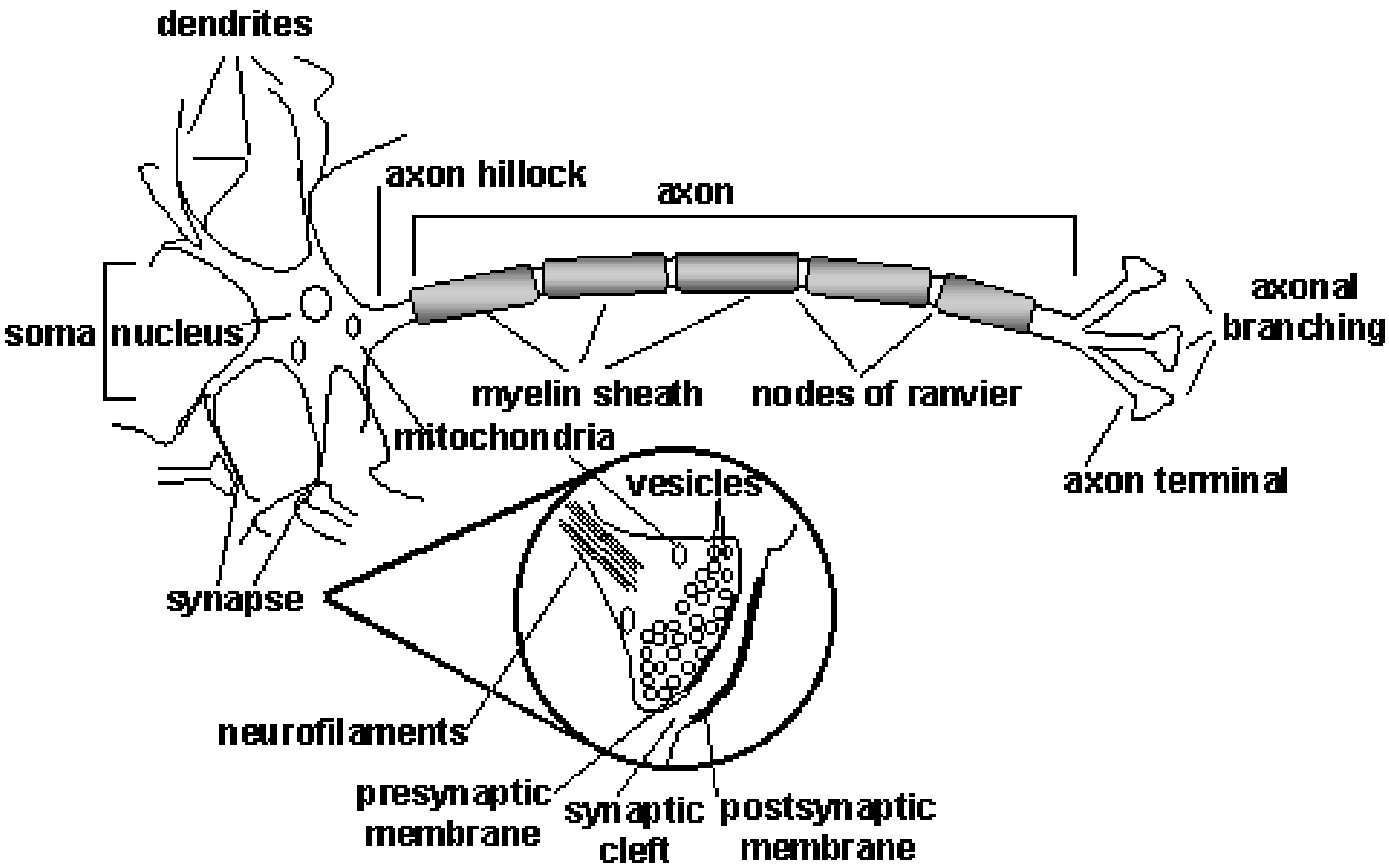
3. The neuron's ***nucleus*** contains the genetic material in the form of DNA. This exists in most types of cells, not just neurons.

4. ***Dendrites*** are fibres which emanate from the cell body and provide the receptive zones that receive activation from other neurons.

5. ***Axons*** are fibres acting as transmission lines that send activation to other neurons.

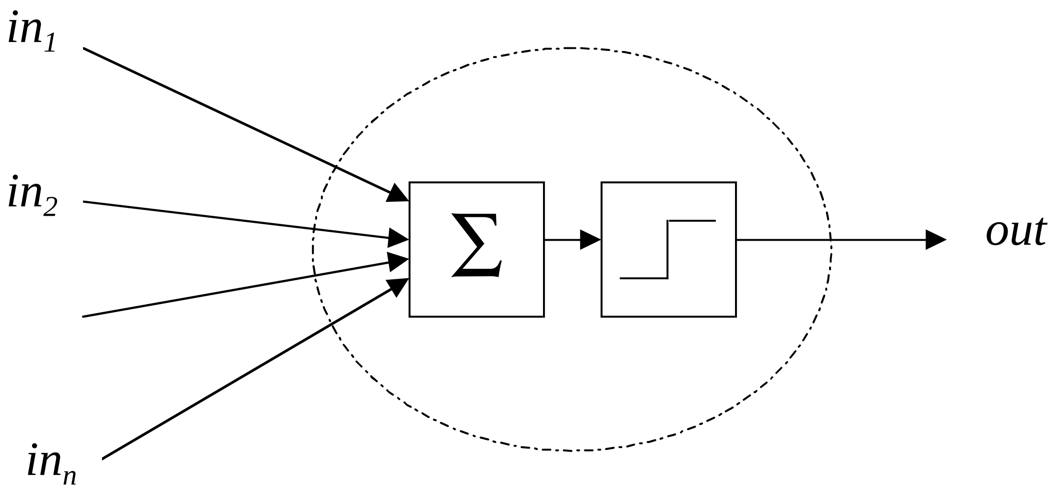
6. The junctions that allow signal transmission between the axons and dendrites are called ***synapses***. The process of transmission is by diffusion of chemicals called ***neurotransmitters*** across the synaptic cleft.

Schematic Diagram of a Biological Neuron



The McCulloch-Pitts Neuron

This vastly simplified model of real neurons is also known as a ***Threshold Logic Unit*** :



1. A set of synapses (i.e. connections) brings in activations from other neurons.
2. A processing unit sums the inputs, and then applies a non-linear activation function (i.e. squashing/transfer/threshold function).
3. An output line transmits the result to other neurons.

Some Useful Notation

We often need to talk about ordered sets of related numbers – we call them ***vectors***, e.g.

$$\boldsymbol{x} = (x_1, x_2, x_3, \ldots, x_n) \quad , \quad \boldsymbol{y} = (y_1, y_2, y_3, \ldots, y_m)$$

The components x_i can be added up to give a ***scalar*** (number), e.g.

$$s = x_1 + x_2 + x_3 + \ldots + x_n = \sum_{i=1}^{n} x_i$$

Two vectors of the same length may be ***added*** to give another vector, e.g.

$$\boldsymbol{z} = \boldsymbol{x} + \boldsymbol{y} = (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n)$$

Two vectors of the same length may be ***multiplied*** to give a scalar, e.g.

$$p = \boldsymbol{x.y} = x_1 y_1 + x_2 y_2 + \ldots + x_n y_n = \sum_{i=1}^{n} x_i y_i$$

To any ambiguity/confusion, we will mostly use the component notation (i.e. explicit indices and summation signs) throughout this module.

The Power of the Notation : Matrices

We can use the same vector component notation to represent complex things with many more dimensions/indices. For two indices we have matrices, e.g. an $m \times n$ ***matrix***

$$\mathbf{w} = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{1n} \\ \vdots & \vdots & & \vdots \\ w_{m1} & w_{m1} & \dots & w_{mn} \end{pmatrix}$$

Matrices of the same size can be ***added*** or ***subtracted*** component by component.

An $m \times n$ matrix $\mathbf{a}$ can be ***multiplied*** with an $n \times p$ matrix $\mathbf{b}$ to give an $m \times p$ matrix $\mathbf{c}$. This becomes straightforward if we write it in terms of components:

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$$

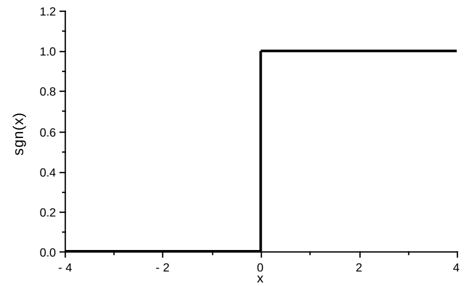
An n component vector can be regarded as a $1 \times n$ or $n \times 1$ matrix.

Some Useful Functions

A function $y = f(x)$ describes a relationship (input-output mapping) from x to y.

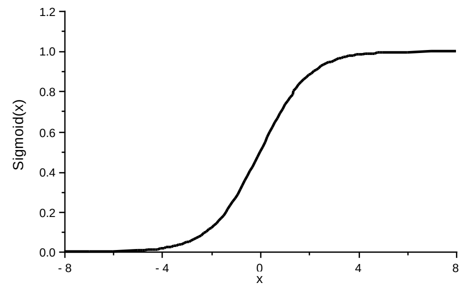
Example 1 The threshold or sign function sgn(x) is defined as

$$\mathrm{sgn}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$



Example 2 The logistic or sigmoid function Sigmoid(x) is defined as

$$\mathrm{Sigmoid}(x) = \frac{1}{1 + e^{-x}}$$



This is a smoothed (differentiable) form of the threshold function.

The McCulloch-Pitts Neuron Equation

Using the above notation, we can now write down a simple equation for the ***output*** *out* of a McCulloch-Pitts neuron as a function of its n ***inputs*** in_i :

$$out = \mathrm{sgn}\left(\sum_{i=1}^{n} in_i - \theta\right)$$

where θ is the neuron's activation ***threshold***. We can easily see that:

$$out = 1 \quad \text{if} \quad \sum_{k=1}^{n} in_k \geq \theta \qquad\qquad out = 0 \quad \text{if} \quad \sum_{k=1}^{n} in_k < \theta$$

Note that the McCulloch-Pitts neuron is an extremely simplified model of real biological neurons. Some of its missing features include: non-binary inputs and outputs, non-linear summation, smooth thresholding, stochasticity, and temporal information processing.

Nevertheless, McCulloch-Pitts neurons are computationally very powerful. One can show that assemblies of such neurons are capable of universal computation.

Overview and Reading

1. Biological neurons, consisting of a cell body, axons, dendrites and synapses, are able to process and transmit neural activation.
2. The McCulloch-Pitts neuron model (Threshold Logic Unit) is a crude approximation to real neurons that performs a simple summation and thresholding function on activation levels.
3. Appropriate mathematical notation facilitates the specification and programming of artificial neurons and networks of artificial neurons.

Reading

1. Haykin: Sections 1.1, 1.2, 1.3
2. Beale & Jackson: Sections 1.2, 3.1, 3.2
3. Gurney: Sections 2.1, 2.2.
4. Ham & Kostanic: Sections 1.2, 1.3