A22648

Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

THE UNIVERSITY OF BIRMINGHAM

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06 00000

Cryptography

May 2016 1.5 hours

[Answer ALL questions]

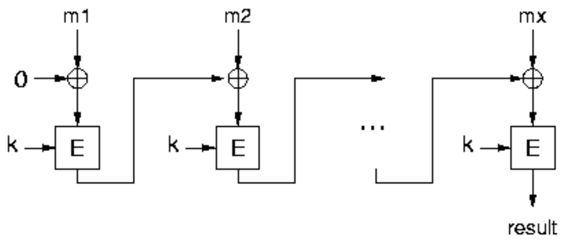
1. Let E be a secure block cipher, such as AES. E takes two arguments: a 128-bit key k, and a single 128-bit message m block. It returns a 128-bit ciphertext block $c = E(k, m)$.

   (a) Explain how to use the secure block cipher E in counter mode (also called CTR mode), in order to securely encrypt a message m that consists of multiple blocks, say $m_1, \ldots, m_n$. In your answer, you should give the definition of the ciphertext blocks $c_1, \ldots, c_n$.
   [10%]

   (b) Does encryption using a secure block cipher in counter mode guarantee message integrity? Explain your answer. **[5%, 10%]**

2. (a) Explain the concept of message authentication code (MAC). Briefly give an example of how it might be used. **[5%]**

   (b) As a reminder, a diagram showing the workings of CBC-MAC over a block cipher E is shown below.



   Suppose a message m consists of two blocks m_1 and m_2. Write down an expression which is the result of computing a CBC-MAC (over the block cipher E and key k) of $m = m_1||m_2$. **[10%]**

   (c) Again, let m_1, m_2 be message blocks, and suppose the attacker possesses just two message-tag pairs, namely (m_1, t_1) and (m_2, t_2).

      (i) Write down t_1 and t_2 in terms of m_1 and m_2 **[3%]**

      (ii) Let $m = m_1||(m_2 \oplus t_1)$. Write down the tag for m in terms of m_1, m_2 and t_1.
      [3%]

      (iii) Explain why this calculation shows that CBC-MAC is not secure without adding further restrictions to how it is used. **[4%]**

3. (a) Explain the concept of semantic security (IND-CPA security) for public key encryption (PKE). Discuss whether a PKE scheme with a deterministic encryption algorithm can be IND-CPA or not. **[5%,5%]**

   (b) Let RSA.PKE = (RSA.Kg,RSA.Enc,RSA.Dec) be the (plain) RSA public key encryption scheme. Let (E,D) be an IND-CPA symmetric encryption scheme using a 256-bit long secret key. Let H be a secure hash function, such as SHA-256.

      (i) Consider RSA.HybPKE = (RSA.Kg,RSA.HybEnc,RSA.HybDec), the combined PKE scheme obtained by defining RSA.HybEnc as follows:
      RSA.HybEnc(PK, m):
      - Choose a random plaintext R from RSA message space
      - Compute $c_0 =$ RSA.Enc(PK, R)
      - Compute $c_1 =$ E$(H(R), m)$
      - Output (c_0, c_1)

      (ii) Describe the corresponding decryption algorithm RSA.HybDec. **[5%]**

      (iii) Is RSA.HybPKE IND-CPA secure in the Random Oracle Model? Discuss your answer. **[5%,5%]**

4. (a) Explain the concept of existential unforgeability for digital signatures. **[8%]**

   (b) What is a *public key certificate* and why do we need them? **[7%]**

   (c) Let us recall the Schnorr digital signature scheme:

   - KG(λ)
     - choose a λ bit prime p and a 256-bit prime q, such that q divides $p-1$ and q is the order of a subgroup $G_q = \langle g \rangle$ of $\mathbf{Z}_p^\star$
     - a cryptographic hash function $H : \{0,1\}^\star \to \{0,1\}^{256}$ (e.g., SHA-256)
     - Choose a random x from $\{0, \ldots, q-1\}$ (i.e. from $\mathbf{Z}_q$)
     - Compute $y = g^x \bmod p$
     - Publish the public key $vk = (p, q, g, y, H)$
     - Retain the private key $sk = x$
   - Sign(sk, M). To sign a bit-string M do:
     - Choose a random r from $\{0, \ldots, q-1\}$
     - Compute $s = H(M \| g^r) \bmod q$
     - Compute $t = (r + x \cdot s) \bmod q$
     - Output signature $\sigma = (s, t)$
   - Verify(vk, σ, m) works as follows:
     - Parse σ as (s, t)
     - Accept the signature if $H(M \| g^t y^{-s}) = s$
     - Otherwise reject the signature

   Let us argue that randomness re-use while signing is insecure. To see this, show that if the same random $r \in \mathbf{Z}_q$ is used to create a signature (s, t) on a message M, and a signature (s', t') on a message M', then an attacker can recover from these two signatures the signing key x, as long as $M \neq M'$.

   [10%]