A22648

Calculators may be used in this examination provided they are not capable of being used to store alphabetical information other than hexadecimal numbers

THE UNIVERSITY OF BIRMINGHAM

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06 00000

Cryptography

May 2017 1.5 hours

[Answer ALL questions]

1. DES is a block cipher with an effective key length of 56 bits (that is, there are 2^{56} distinct keys). 3DES is a block cipher defined in terms of DES. 3DES uses three DES keys k_1, k_2 and k_3. The encryption c of a block m using 3DES is given by $c = \mathsf{Enc}_{k_1}(\mathsf{Dec}_{k_2}(\mathsf{Enc}_{k_3}(m)))$.

   (a) Explain why 3DES is better than DES. **[7%]**

   (b) Explain why encryption with 3DES is defined as $\mathsf{Enc}_{k_1}(\mathsf{Dec}_{k_2}(\mathsf{Enc}_{k_3}(m)))$ and not $\mathsf{Enc}_{k_1}(\mathsf{Enc}_{k_2}(\mathsf{Enc}_{k_3}(m)))$. **[7%]**

   Joe is considering to use "2DES", which he defines using

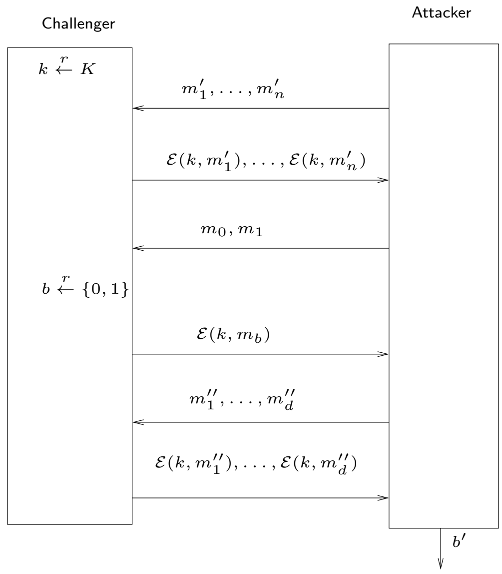
   2DES: $$c = \mathsf{Enc}_{k_1}(\mathsf{Enc}_{k_2}(m))$$

   (it uses only two DES keys). He hopes that this will give him approximately twice the bitlength security of DES.

   (c) Show that the equation (2DES) above may be equivalently written $\mathsf{Enc}_{k_2}(m) = \mathsf{Dec}_{k_1}(c)$, which has no nested Enc or Dec funcions. **[6%]**

   (d) Suppose an attacker has a plaintext message m and the corresponding ciphertext c. Explain how it can find the keys k_1 and k_2 used by Joe, using a little more than 2^{56} operations (and way less than Joe's expected 2^{112} operations). **[5%]**

2. The figure below is to remind you of the IND-CPA game.



(a) What is the condition on the output b' that indicates that the attacker has won the game? **[6%]**

For each of the following definitions of $\mathcal{E}(k, m)$ explain whether the attacker can expect to win the game.

(b) $\mathcal{E}(k, m)$ is defined as the encryption of a message m using AES in counter mode, the key k, and a randomly-chosen IV. **[7%]**

(c) $\mathcal{E}(k, m)$ is defined as the HMAC of a message m using SHA-256 as the underlying hash function, and the key k. **[7%]**

You are designing a system which will encrypt and save some information m on a disk, and later retrieve and decrypt the information. You want to use *authenticated encryption*. You have an encryption function $\mathcal{E}$ satisfying IND-CPA and a MAC function $\mathcal{M}$ satisfying the MAC unforgeability game, and two secret keys k_1 and k_2.

(d) Which of the following ways to do it is better?

  (i) Encrypt-then-MAC: encrypt the message, then compute MAC of ciphertext. The result may be written $\mathcal{E}(k_1, m)$, $\mathcal{M}(k_2, \mathcal{E}(k_1, m))$.

  (ii) Encrypt and MAC: The result consists of the encryption of m and the MAC of m, and may be written $\mathcal{E}(k_1, m)$, $\mathcal{M}(k_2, m)$.

  Explain your answer. **[4%]**

3. Let us consider **RSA-D** = (Kg,Enc,Dec), a variant of the RSA public key encryption scheme.

   - Key generation KG(λ)
     - Generate two distinct odd primes p and q of same bit-size λ
     - Compute $N = p \cdot q$ and $\phi = (p-1)(q-1)$
     - Select two random integers $1 < e_1, e_2 < \phi$ such that $\gcd(e_1, \phi) = 1$ and $\gcd(e_2, \phi) = 1$
     - Compute the unique integer $1 < d_1 < \phi$ such that $e_1 \cdot d_1 \equiv 1 \bmod \phi$
     - Compute the unique integer $1 < d_2 < \phi$ such that $e_2 \cdot d_2 \equiv 1 \bmod \phi$
     - The public key is $PK = (N, e_1, e_2)$. The private key is $SK = (d_1, d_2)$
   - Encryption Enc(PK, m) a message $m \in \mathbf{Z}_N^\star$ proceeds as follows:
     - Generate a random integer r in $\mathbf{Z}_N^\star$
     - Compute $c_1 = r^{e_1} \bmod N$
     - Compute $c_2 = m^{e_2} \cdot r^{e_1 \cdot e_2} \bmod N$
     - Output $C = c_1 || c_2$

   (a) Give the corresponding decryption algorithm Dec(SK, C). Prove your decryption algorithm is correct, i.e. that given a legitimate key pair (PK, SK) it holds Dec$(SK, \text{Enc}(PK, m)) = m$ for any admissible m. **[10%]**

   (b) Let us study the security of the asymmetric encryption scheme **RSA-D**:

      (i) Describe in technical terms what the statement *"Breaking the RSA problem is hard"* means. **[5%]**

      (ii) What is the definition of *one-wayness* for a public key encryption scheme? **[5%]**

      (iii) Is **RSA-D** a one-way secure public key encryption scheme? Justify your answer. **[5%]**

      (iv) Is **RSA-D** an IND-CPA (or semantically secure) public key encryption scheme? Justify your answer. **[5%]**

4. Let us consider ElGamal encryption parameters (G, g, p, q) for large primes p, q, where g is a generator of a subgroup G of $\mathbf{Z}_p^\star$ and G has q elements. Recall that in ElGamal every user chooses a random private key $x \in \mathbf{Z}_q$ and computes the public key $X = g^x \bmod p$. To encrypt a message $m \in G$ for a user with public key X, the sender chooses a random $y \in \mathbf{Z}_q$ and computes the ciphertext $(g^y, X^y \cdot m)$.

   (a) How is ElGamal encryption related to the Diffie-Hellman key exchange protocol? Describe the Diffie-Hellman (DH) protocol in detail. **[5%]**

   (b) Why is DH key exchange insecure against man-in-the-middle (MitM) attacks? Describe a MitM attack against the DH protocol in detail. **[5%]**

   (c) Present an improvement of the DH key exchange that prevents MitM attacks. Explain your answer. **[10%]**