

Answers to Simple proofs in PL

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Some Logic Problems

1. Phillip and Quinton watch football. If Quinton watches football then Roger watches hockey.
Therefore Roger watches hockey.

P = Phillip watches football

Q = Quinton watches football

R = Roger watches hockey therefore we can translate the above into

$P \ \& \ Q, \ Q \rightarrow R : R$

1. P & Q Premise {1}

2. Q &-Elim {1}

3. Q \rightarrow Z Premise {3}

4. Z Modus Ponens {1,3}

2. P, Q, R : (P & Q) & R

1. P Premise {1}

2. Q Premise {2}

3. R Premise {3}

4. P & Q & Intro {1,2}

5. (P & Q) & R & Intro {1,2,3}

3. P & \sim Q, R \rightarrow Q, \sim R \rightarrow Z : Z

1. P & \sim Q Premise

2. \sim Q &-elim {1}

3. R \rightarrow Q Premise {3}

4. \sim R Modus Tollens {1,3}

5. \sim R \rightarrow Z Premise {5}

6. Z Modus Ponens {1,3,5}

4. R, P \leftrightarrow Q, Q \rightarrow Z, R \rightarrow P : Z

1. P \leftrightarrow Q Premise {1}

2. (P \rightarrow Q) & (Q \rightarrow P) Biconditional {1}

3. P \rightarrow Q &elim {1}

4. R Premise {4}

5. R \rightarrow P Premise {5}

6. P Modus Ponens {5, 4}

7. Q Modus Ponens {5,4,1}

8. Q \rightarrow Z Premise {8}

9. Z Modus Ponens {5,4,8}

5. $(\sim P \rightarrow \sim Q), (P \rightarrow Z), \sim\sim Q : Z$

1. $\sim\sim Q$ Premise {1}

2. $\sim P \rightarrow \sim Q$ Premise {2}

3. $\sim\sim P$ Modus Tollens {1,2}

4. P Double Negative Elim {1,2}

5. $P \rightarrow Z$ Premise {5}

6. Z Modus Ponens {1,2, 5}