Language and Logic

Classwork 10: From English to Predicate Logic

"All philosophers have a beard. Some logicians have beards too. No philosopher is also a logician. Alan is a philosopher. Bill and Charlie are logicians. Charlie doesn't have a beard but Bill has a beard. Alan either has a beard or doesn't have one. David is either a philosopher or a logician"

a.
Identify and represent every statement in the passage above in predicate logic.

Domain Academics
P ... is a philosopher
L ... is a logician
B ... has a beard
$\forall x\,[Px \rightarrow Bx]$
$\neg\, \exists x\,[Px \wedge Lx]$
Pa
$Lb \wedge Lc$
$\neg Bc \wedge Bb$
$Ba \vee \neg Ba$
$Pd \vee Ld$

8%

b. Provide a predicate logic expression which states that if somebody is either a philosopher or logician then they understand logic.

$\forall x[(Px \vee Lx) \rightarrow Ux]$

U ... understand logic.

2%

c. For an academic to understand their own field, they must first study it. Write an expression which captures the idea that if somebody is either a logician or philosopher then they must have first studied either logic or philosophy respectively.

$\forall x[Px \rightarrow SPx] \wedge \forall x[Lx \rightarrow SLx]$

SP ... has studied philosophy

SL ... has studied logic

I suspect quite a few will get the conditionals backwards (e.g. SPx → Px)

5%

d. Prove either David has a beard or he's a logician.

$Pd \therefore Ld \vee Pd, \forall x\,[Px \rightarrow Bx] : Bd \vee Ld$

1. $Pd \vee Ld$	Premise	{1}
2. | Pd	H	{2}
3. | $\forall x\,[Px \rightarrow Bx]$	Premise	{2,3}
4. | $Pd \rightarrow Bx$	UE 2,3	{2,3}
5. | Bd	→E 2,4	{2,3}
6. | $Bd \vee Ld$	VI 5	{2,3}
7. | Ld	H	{7}

8. | Bd v Ld VI 7 {7}
9. Bd v Ld VE 1,6,8 {1.3}

[10%]

Question 2
John, Paul and Matthew are all possible suspects. John was seen at the crime scene and has a motive. Paul has an alibi. Matthew does not have an alibi. John either has an alibi or does not. Anybody who is guilty is not innocent.

1. Identify all the propositions in the above text and for each proposition, express them in predicate logic by first providing a domain and then writing a predicate logic expression for each proposition.
[10%]

D: Crime Scene
S = ... is a suspect
C = ... seen at the crime scene
M = ... has a motive
A = ... has an alibi
G = ... is guilty
I = ... is innocent

j = john
p = paul
m = matthew

Sj ∧ Sp ∧ Sm
Cj ∧ Mj
Ap
¬Am
Aj v ¬Aj
Vx [Gx → ¬Ix]

2. Provide a rule which states that if anybody has an alibi then they are innocent.
[2%]

I ... is innocent
Vx [Ax → Ix]

3. Prove using natural deduction that at least somebody is innocent
[5%]

1. Vx [Ax → Ix] Premise
2. Ap → Ip UE
3. Ap (Premise)
4. Ip IE {2,3}
5. ∃x[Ix] EI {4}

4. John is found to be guilty (he confesses). Using all the propositions you have provided, prove that it is therefore impossible that John had an alibi.
[8%]

1. | *Aj H*
2. | *Vx [Ax → Ix] Premise*
3. | *Aj → Ij UE {2}*
4. | *Ij IE {3,1}*
5. | *Gj Premise*
6. | *Vx [Gx → ¬Ix]*
7. | *Gj → ¬Ij UE {6}*
8. | *¬Ij IE {7, 5}*
9. *¬Aj RAA {8, 4, 1}*