

# Natural Deduction in Propositional Logic

## Classwork

### 1. A simple problem

$P, P \rightarrow Q : P \wedge (Q \vee R)$

1. P Premise {1}
2.  $P \rightarrow Q$  Premise {2}
3. Q E $\rightarrow$  1,2 {1,2}
4.  $Q \vee R$  Iv 3 {1,2}
5.  $P \wedge (Q \vee R)$  1, 4 I $\wedge$  {1,2}

### 1. An absurd problem

$P \rightarrow Q, \sim Q : \sim P$

1.  $P \rightarrow Q$  Premise {1}
2.  $\sim Q$  Premise P {2}
3. | P H {3}
4. | Q E $\rightarrow$  1,3 {1,3}
5. |  $\sim Q$  It 2{2}
6.  $\sim P$  RAA 3,4,5 {1,2}

### 2. Suppose the contrary

$:(P \wedge \sim P)$

1. |  $P \wedge \sim P$  H {1}
2. | P E $\wedge$  1 {1}
3. |  $\sim P$  E $\wedge$  1 {1}
4.  $\sim(P \wedge \sim P)$  1,2,3 I $\sim$  { }

### 3. Implications

$P \rightarrow (Q \rightarrow R) : Q \rightarrow (P \rightarrow R)$

1.  $P \rightarrow (Q \rightarrow R)$  Premise {1}
2. | Q H {2}
3. | | P H {3}
4. | |  $Q \rightarrow R$  E $\rightarrow$  1,2,3 {1,2,3}
5. | | R E $\rightarrow$  2,4 {1,2,3}
6. |  $P \rightarrow R$  I $\rightarrow$  3, 5 {1,2}
7.  $Q \rightarrow (P \rightarrow R)$  I $\rightarrow$  2,6 {1}

## 5 A tough problem

$A \vee B, A \rightarrow C, \sim D \rightarrow \sim B : C \vee D$

1.  $A \vee B$  Premise {1}
2.  $A \rightarrow C$  Premise {2}
3.  $\sim D \rightarrow \sim B$  Premise {3}
4. A H {4}
5. |  $C$  E- $\rightarrow$ 2,4 {2,4}
6. |  $C \vee D$  Iv 5{2,4}
7. B H {7}
8. |  $\sim D$  H {7,8}
9. | |  $\sim B$  E- $\rightarrow$  3, 8 {7,8,3}
10. | | B It 7 {7}
11. |  $\sim \sim D$  RAA 7,9,10 {7,3}
12. | D E~ 11 {7,3}
13. |  $C \vee D$  Iv 12 {7,3}
14.  $C \vee D$  Ev 1,6,13 {1,3}