

Theorem Introduction

Language and Logic

Classwork 6

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1. Warm up.

$P \rightarrow Q, Q \rightarrow Z : P \rightarrow Z$

1. $\underline{P} H \{1\}$
2. $\mid P \rightarrow Q$ Premise $\{1,2\}$
3. $\mid Q E \rightarrow 1,2 \{1,2\}$
4. $\mid Q \rightarrow Z$ Premise $\{1,4\}$
5. $\mid Z E \rightarrow 3,4 \{1,2,4\}$
6. $P \rightarrow Z 1,5 \{2,4\}$

2. Excluded middle (You'll need this for the homework)

$: P \vee \neg P$

1. $\mid \underline{\neg(P \vee \neg P)} H \{1\}$
2. $\mid \underline{P} H \{1,2\}$
3. $\mid \mid P \vee \neg P$ Iv 2 $\{1,2\}$
4. $\mid \mid \neg(P \vee \neg P) i \{1\}$
5. $\mid \neg P$ RAA 2,3,4 $\{1\}$
6. $\mid \underline{\neg P} H \{1,6\}$
7. $\mid \mid P \vee \neg P$ Iv $\{1, 6\}$
8. $\mid \mid \neg(P \vee \neg P) it 1$
9. $\mid \neg \neg P$ I- 6,7,8 $\{1\}$
10. P DNE 9 $\{1\}$
11. $\neg \neg(P \vee \neg P)$ RAA 1, 5, 10 $\{\}$
12. $P \vee \neg P$ DNE 11 $\{\}$

There is a shorter proof - it's not nearly as satisfying though.

3. Theorem Introduction

$P \rightarrow Q, \neg P \rightarrow Q: Q$

1. $P \rightarrow Q$ Premise $\{1\}$
2. $\neg P \rightarrow Q$ Premise $\{2\}$
3. $P \vee \neg P$ TI $\{\}$ (Question 2)
4. $\underline{P} H \{4\}$
5. $\mid Q E \rightarrow 1,4 \{1,4\}$
6. $\underline{\neg P} H \{6\}$
7. $\mid Q E \rightarrow 2,6 \{2,6\}$
8. Q Ev 3, 4, 5, 6, 7 $\{1,2\}$

4. not P and not P

$: \neg(P \wedge \neg P)$

1. $\underline{P \wedge \neg P} H \{1\}$
2. $\mid P E \wedge \{1\}$
3. $\mid \neg P E \wedge \{1\}$
4. $\neg(P \wedge \neg P)$ RAA 1,2,3 $\{\}$