Translating Predicate Logic

Classwork 7

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Recap

Last week we introduced Predicate Logic and the use of ∀ and ∃. As a reminder:

> ∀x [Fx] = For all values of x, F is true of x
> ∃x [Fx] = There exists some x where F is true of that x
>
> Fx where F is some property such as "... likes cheese"

We also presented two general rules for transforming natural language into predicate logic.

> Universal Conditional
> All As are Bs → ∀x [Ax → Bx]
>
> Existentially qualified conjunction
>
> Some As are Bs → ∃x [Ax ∧ Bx]

Negation

Negation has a special importance in predicate logic. Consider how to represent:

There are no unicorns	¬∃x [Ux]	∀x [¬Ux]

Translating Natural Language into Predicate Logic

Something is A	∃x [Ax]	¬∀x[¬Ax]
Nothing is A	¬ ∃x [Ax]	∀x [¬Ax]
Everything is A	∀x [Ax]	¬∃x [¬Ax]
Some As are Bs	∃x [Ax ∧ Bx]	¬∀x [¬(Ax ∧ Bx)]
All As are Bs	∀x [Ax → Bx]	¬∃x [Ax ∧ ¬Bx]
No As are Bs	¬∃x [Ax ∧ Bx]	∀x [Ax → ¬Bx]

Relationships 1

Convert the following into predicate logic:

I Domain(human relationships)
L = ... loves ....
H = ...hates ....
j = John
m = Mary
c = Chris

John loves Mary Ljm

Everybody hates Chris $\forall x$ [Hxc]

Somebody loves Chris $\exists x$ [Lxc]

John loves everybody $\forall x$ [Ljx]

John loves somebody $\exists x$ [Ljx]

Nobody loves John $\neg\exists x$ [Lxj] or $\forall x$ [$\neg$Lxj]

Mary doesn't love anybody ∧ John loves Mary $\neg\exists x$ [Lmx] ∧ Ljm

Mary hates Chris but Chris loves Mary Lmc ∧ Lcm

Mary doesn't love everybody or somebody doesn't love Mary

$\forall x$ [$\neg$Lmx] ∨ $\exists y$ [$\neg$Lym]

If Mary loves everybody then somebody doesn't love Mary and Mary loves somebody

$\forall x$ [Lmx] → $\exists y$ [$\neg$Lym] ∧ $\exists z$ [Lmz]

Relationships 2

Everybody loves everybody $\forall x\ [\ \forall y\ [\ Lxy]]$

Everybody loves somebody $\forall x\ [\ \exists y\ [\ Lxy]]$

Everyone loves themselves $\forall x\ [Lxx]$

Everybody loves anybody with red hair

$$\forall x\ [\ \forall y[\ Ry \rightarrow Lxy]]$$

All Virgos love Taurens

$$\forall x\ [\ \forall y\ [Vx \wedge Ty \rightarrow Lxy]]$$

All Virgos love a Taurens

$$\forall x\ [\ \exists y\ [Vx \wedge Ty \rightarrow Lxy]]$$

The Cosmological Argument

Thomas Aquinas (1225-1274) originally proposed the cosmological argument for the existence of God (though he was inspired by Plato). The argument goes (roughly) as follows:

1) There are things in this world that are contingent – they might not have existed e.g. we would not exist without our parents
2) All things in the world are like this – everything depends on something else for its existence
3) Therefore there must be a cause of everything in the universe that exists outside of it
4) This cause must be a necessary being – one which contains the reason for its existence inside itself
5) This necessary being is God

This version of the argument was proposed by Coplestone during a radio debate with Bertrand Russell in 1947. Russell's later reply used logic to argue that the above argument was invalid.

Russell proposed the following as an analogy to the argument:

Everybody has a mother therefore somebody is the mother of everyone. If we suppose that

M = ... is the mother of ...

then one translation of "Everybody has a mother" could be

$$\forall x\ [\ \exists y\ [\ Mxy\ ]\ ]$$

However this is wrong – and if you analyse it carefully this really says "Everybody is the mother of someone". Rather the correct translation is

$$\forall x\ [\ \exists y\ [Myx]\ ]$$

The second part of the argument translates as

$$\exists y\ [\ \forall x\ [Myx]]$$

and therefore $\forall x\ [\ \exists y\ [Myx]\ ] : \exists y\ [\ \forall x\ [Myx]]$

Intuitively this looks like an invalid and in fact is an example of what is called the "quantifier switch" fallacy. We'll talk more about fallacies at the end of term. However I leave it up to you (and your own opinions) about whether the Cosmological argument is valid or invalid.