

# Inference in Predicate Logic

Classwork

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## 1. Warm up

“Arthur is a raven who is not black. Therefore not all ravens are black.”

### I (Ravens)

R = ... is a raven

B = ... is black

a = ... arthur (a raven)

$Ra \ \& \ \sim Ba : \sim \forall x [Rx \rightarrow Bx]$

1. |  $\forall x [Rx \rightarrow Bx]$     H {1}
2. |  $Ra \rightarrow Ba$         UE {1}
3. |  $Ra \ \& \ \sim Ba$         Premise {1,3}
4. |  $Ra$                     &E {1,3}
5. |  $Ba$                      $\rightarrow$ E {1,3}
6. |  $\sim Ba$                 &E {1,3}
7.  $\sim \forall x [Rx \rightarrow Bx]$     RAA {3}

## 2. A question of European identity

All Flemish are from Belgium. All Belgians are European. Therefore All Flemish are European.

F = is Flemish

B = is from Belgium

E = is European

$\forall x [Fx \rightarrow Bx], \ \forall x [Bx \rightarrow Ex] : \forall x [Fx \rightarrow Ex]$

1.  $\forall x [Fx \rightarrow Bx]$         Premise {1}
2.  $\forall x [Bx \rightarrow Ex]$         Premise {2}
3. | Fa                    Hypothesis {3}
4. |  $Fa \rightarrow Ba$         UE {1}        {3,1}
5. |  $Ba$                      $\rightarrow$ E {3,1}
6. |  $Ba \rightarrow Ea$         UE {3,2}
7. |  $Ea$                      $\rightarrow$ E {3,2,1}
8.  $Fa \rightarrow Ea$                      $\rightarrow$ I {1,2}
9.  $\forall x [Fx \rightarrow Ex]$         UI {1, 2} This is valid because it depends only on the initial universals

## 3. A choice exists

$\forall x [Fx] : \exists x [Fx \vee Gx]$

1.  $\forall x [Fx]$                 Premise {1}
2.  $Fa$                     UE {1}
3.  $Fa \vee Ga$                  $\vee$ I {1}
4.  $\exists x [Fx \vee Gx]$         EI {1}

## 4. An example to them all

$\exists x [Px \rightarrow Qx], \forall x [Px] : \exists x [Qx]$

1.  $\exists x [Px \rightarrow Qx]$  Premise {1}
2. |  $Pa \rightarrow Qa$  H {2}
3. |  $\forall x [Px]$  Premise {2,3}
4. |  $Pa$  UE {2,3}
5. |  $Qa$   $\rightarrow$ E {2,3}
6. |  $\exists x [Qx]$  EI {2,3}
7.  $\exists x [Qx]$  EE {1,3}

5 Be careful

$\forall x [Fx \rightarrow (Hx \& Jx)] \forall x [\sim(Hx \& Jx)] : \forall x [\sim Fx]$

1.  $\forall x [Fx \rightarrow (Hx \& Jx)]$  Premise {1}
2.  $Fa \rightarrow (Ha \& Ja)$  UE1 {1}
3.  $\forall x [\sim(Hx \& Jx)]$  Premise {3}
4.  $\sim(Ha \& Ja)$  UE3 {3}
5. |  $Fa$  H {5}
6. |  $Fa \rightarrow Ha \& Ja$  i 2 {1}
7. |  $Ha \& Ja$   $\rightarrow$ E {1,5}
8. |  $\sim(Ha \& Ja)$  i 4 {3}
9.  $\sim Fa$  RAA 5,7,8 {1,3}
10.  $\forall x [\sim Fx]$  UI {1,3}