Planning Proofs in Predicate Logic

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Classwork

1.A moral tale

Some rich men are kind. All kind men are loved. Therefore, some rich men are loved.

R = is rich
K = is kind
L = is loved
a = adam

∃x [Rx & Kx], ∀x [Kx → Lx] : ∃x [Rx & Lx]

1. ∃x [Rx & Kx]   Premise {1}
2. | Ra & Ka   H {1, 2}
3. | ∀x [Kx → Lx]   Premise {1, 2,3}
4. | Ka → La   UE 3 {1, 2,3}
5. | Ka   &E 2 {1, 2}
6. | La   →E 4,5 {1, 2,3}
7. | Ra   &E 2 {1, 2}
8. | Ra & La   &I 6,7 {1, 2,3}
9 | ∃x [Rx & Lx]   EI 8 {1, 2,3}
10 ∃x [Rx & Lx]   EE 1, 9 {1,3}

1. A humble beginning

(A & B) → C, B : A → C

1. | A   H   {1}
2. | (A & B) → C   Premise   {1,2}
3. | B   Premise   {1.3}
4. | A & B   &I 1,3   {1,3}
5. | C   →E 2, 4   {1,2,3}
6. A → C   →I 1,5   {2,3}

As a revision aid you might consider proving the above using a truth table.

2. Big Brother

∀x [(Ax & Bx) → Cx], Ba : ∃x [Ax → Cx]

1. ∀x [(Ax & Bx) → Cx]   Premise   {1}
2. Aa & Ba → Ca   UE 1   {1}
3. | Aa   H   {3}
4. | Ba   Premise   {3,4}
5. | Aa & Ba   &I 3,4   {3,4}
6. | Ca   →E 2, 5   {1,3,4}
7. Aa → Ca   → I 3, 6   {1,4}
8. ∃x [Ax → Cx]   EI {7}   {1,4}

Question: if the existential in the conclusion was an universal – would the sequent be valid?

4. No short cuts

$\forall y\ [Gy \rightarrow Hy] : \exists x\ [Gx] \rightarrow \exists z\ [Hz]$

1 | $\underline{\exists x\ [Gx]}$	H	{1}
2 | | $\underline{Ga}$	H	{1,2}
3 | | $\forall y\ [Gy \rightarrow Hy]$	Premise	{1,2,3}
4 | | $Ga \rightarrow Ha$	UE 3	{1,2,3}
5 | | Ha	→E 2,4,5	{1,2,3}
6 | | $\exists z\ [Hz]$	EI 5	{1,2,3}
7 | $\exists z\ [Hz]$	EE 6	{1,3}
8 $\exists x\ [Gx] \rightarrow \exists z\ [Hz]$	→I 1, 7	{3}

(Compare this to $\forall y\ [Gy \rightarrow Hy] : \exists x\ [Gx \rightarrow Hx]$ or even the propositional version)