

Language and Logic  
Exercise 8  
Model Answers  
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This exercise should be very easy for all students. The intention is for them to understand how to “unpack” quantified statements into statements about particular individuals, do some very simple reasoning and then translate back into quantified statements.

1.  $\forall x [Bx] : \exists x[Bx]$

- |                     |             |
|---------------------|-------------|
| 1. $\forall x [Bx]$ | Premise {1} |
| 2. $Ba$             | UE 1 {1}    |
| 3. $\exists x[Bx]$  | EI 2 {1}    |

[2]

2.  $\forall x[Bx \wedge Cx] : \exists x[Bx] \wedge \exists x[Cx]$

- |   |                          |
|---|--------------------------|
| 1. $\forall x[Bx \wedge Cx]$            | Premise {1}              |
| 2. $Ba \wedge Ca$                       | UE 1 {1}                 |
| 3. $Ba$                                 | $\wedge$ _elim 2 {1}     |
| 4. $\exists x[Bx]$                      | EI 3 {1}                 |
| 5. $Ca$                                 | $\wedge$ _elim 2 {1}     |
| 6. $\exists x[Cx]$                      | EI 5 {1}                 |
| 7. $\exists x[Bx] \wedge \exists x[Cx]$ | $\wedge$ _intro 4, 6 {1} |

[2]

3.  $Pa, Qa : \exists x[Px \wedge Qx]$

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|------------------------------|---------------------------|
| 1. $Pa$                      | Premise {1}               |
| 2. $Qa$                      | Premise {2}               |
| 3. $Pa \wedge Qa$            | $\wedge$ _intro 1,2 {1,2} |
| 4. $\exists x[Px \wedge Qx]$ | EI 3 {1,2}                |

[2]

4.  $\forall x[Fx \rightarrow (Gx \rightarrow Hx)], (Fa \wedge Ga) : \exists x[Hx]$

- |  |                              |
|--|------------------------------|
| 1. $\forall x[Fx \rightarrow (Gx \rightarrow Hx)]$ | Premise {1}                  |
| 2. $Fa \rightarrow (Ga \rightarrow Ha)$            | UE 1 {1}                     |
| 3. $Fa \wedge Ga$                                  | Premise {3}                  |
| 4. $Fa$  | $\wedge$ _elim 3 {3}         |
| 5. $Ga \rightarrow Ha$                             | $\rightarrow$ elim 2,4 {1,3} |
| 6. $Ga$  | $\wedge$ _elim 3 {3}         |
| 7. $Ha$  | $\rightarrow$ elim 5,6 {1,3} |
| 8. $\exists x[Hx]$                                 | EI 7 {1,3}                   |

[4]