Dependency Quantified Boolean Formulas

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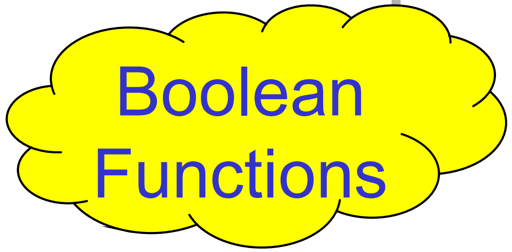
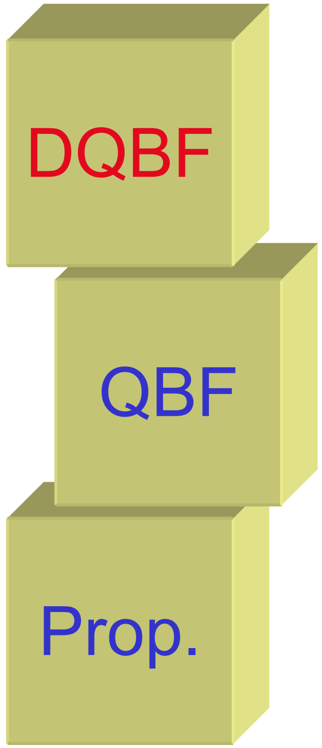
QBF and Beyond



Quantified Boolean Formulas (QBF) extend propositional logic with quantifiers over atoms.

Further extension:
quantifiers with explicit dependencies
$\rightarrow$ Dependency QBF (DQBF)



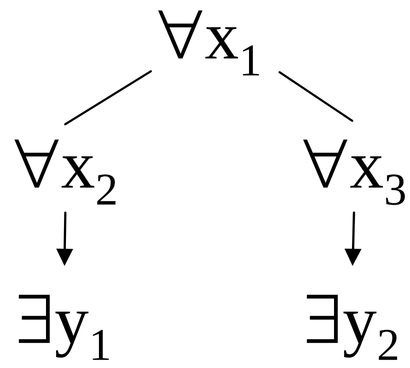
Motivations:

- short representations
- elegant/natural modeling

Partially-Ordered Quantifiers 1/2

Consider a quantified formula where:

- y_1 depends on x_1 and x_2
- y_2 depends on x_1 and x_3



Problem:

By definition, QBFs are in prenex form with a linear order of quantifiers.

$\rightarrow$ we must flatten the quantifier hierarchy

$\forall x_1 \forall x_2 \exists y_1 \forall x_3 \exists y_2\, \phi(x_1,x_2,y_1) \wedge \psi(x_1,x_3,y_2)$

now: y_2 depends on x_1,x_2,x_3

$\rightarrow$ loss of information, larger search space

Partially-Ordered Quantifiers 2/2

Solution:

Partially-ordered Quantification

[Henkin `61; Peterson et al. `79]

→ Dependency Quantified Boolean Formulas

$$\forall x_1 \forall x_2 \forall x_3\, \exists y_1(x_1,x_2)\, \exists y_2(x_1,x_3)\, \phi(x_1,x_2,y_1) \wedge \psi(x_1,x_3,y_2)$$

Ability to indicate for each existential variable on which universals it depends.

Advanced Dependencies

No strict correspondence between dependencies and formula structure required.

Example:

$\forall x_1 \forall x_2 \forall x_3 \exists y_1(x_1,x_2) \exists y_2(x_1,x_3)$

$\phi(x_1,x_2,y_1) \wedge \psi(x_1,x_3,y_2) \wedge \tau(y_1,y_2)$

not possible in (non-prenex) QBF!

Why combine „unrelated“ variables in one subformula?

allows to impose additional restrictions on the choice of existential variables.

$\rightarrow$ increases expressive power

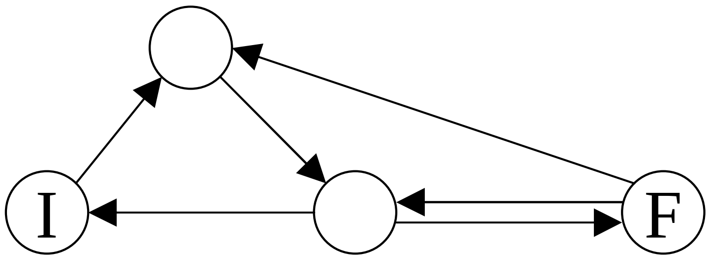
$\rightarrow$ new modeling approaches

Research Agenda 1/2

DQBF Modeling

How can we leverage the increased expressiveness of DQBF for reducing formula sizes?

Modeling Example: Graph Reachability



Is there a path of length k from an initial state to a final state?

$\rightarrow$ DQBF modeling as three-player game.

$\rightarrow$ Only $O(\log k)$ variables vs. $O(k)$ in QBF.

Research Agenda 2/2

DQBF Complexity

DQBF-SAT is known to be NEXPTIME-complete (vs. PSPACE-completeness of QBF).

Can we identify easier/tractable subclasses?

Dependency Quantified Horn Formulas:

$\rightarrow$ can be solved in time $O(|\forall| \cdot |\Phi|)$ (same as QHORN).

$\rightarrow$ can be transformed into equivalent QHORN formulas of length $O(|\forall| \cdot |\Phi|)$.