Pareto Optimality

Introduction to Evolutionary Computation

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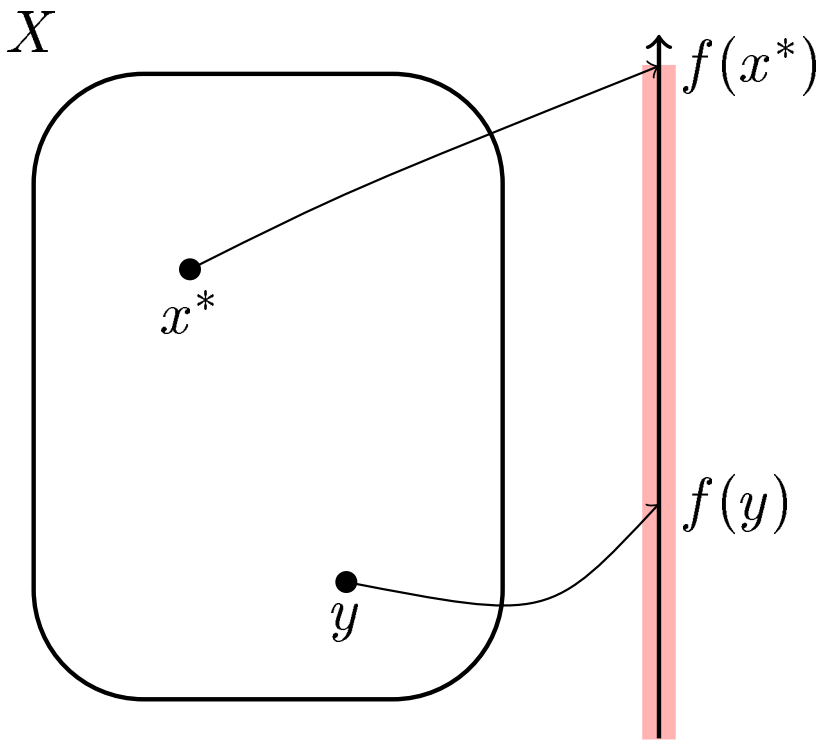
October 26th 2009

Course Work I in Intro to EC (22753)

Deadline This Week

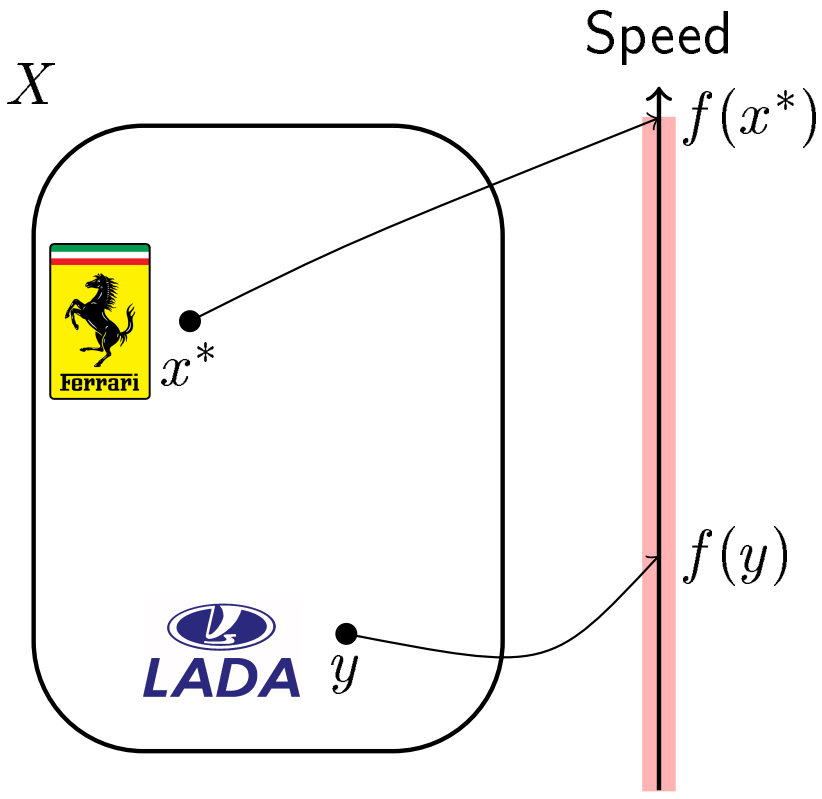
- Course Work I: Function Optimisation.
- Due: October 30, 2009: 12pm.
- Contributes 10% to your final mark.
- Online submission using the school's BOSS system.

Single-objective Optimisation



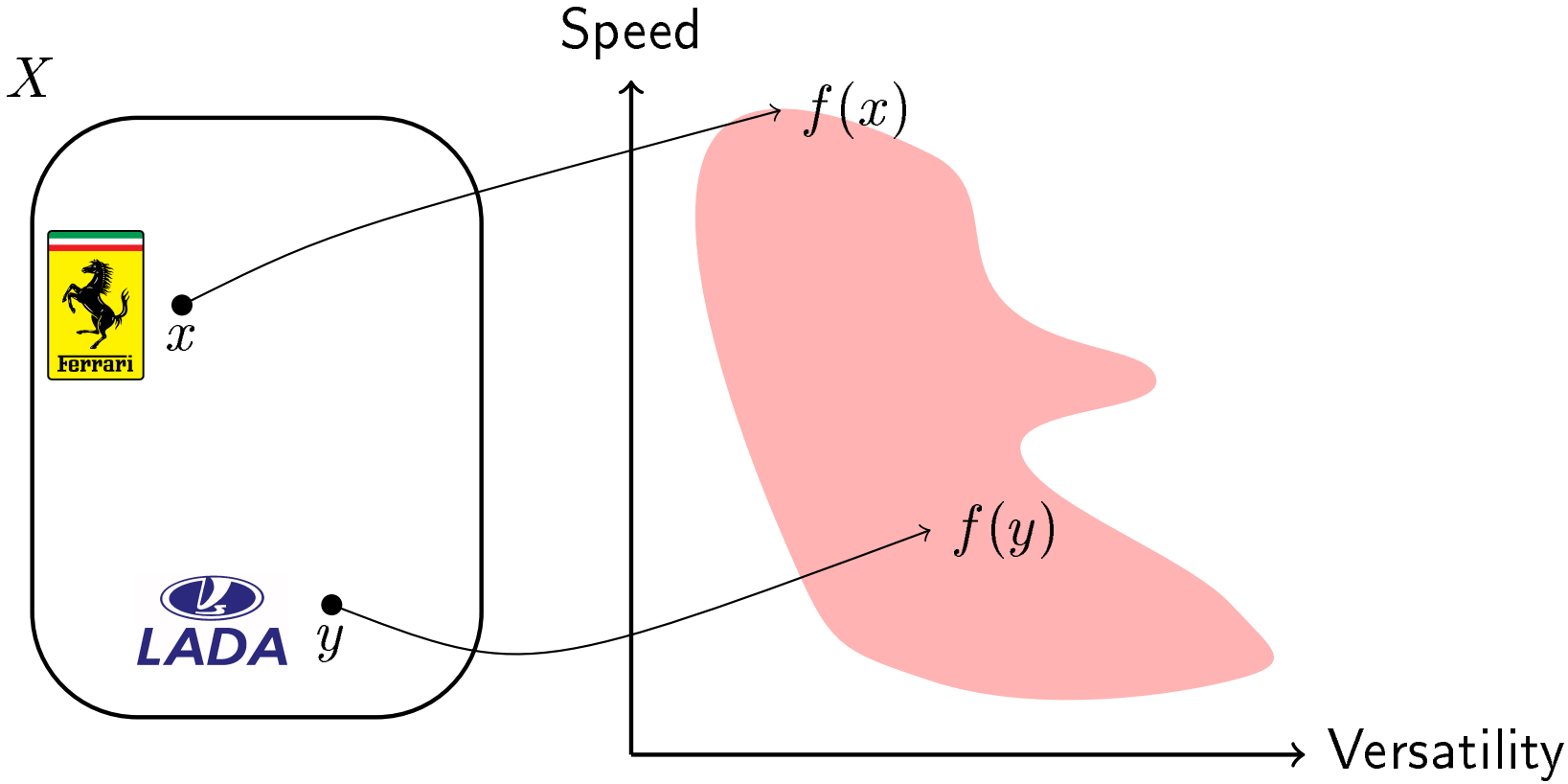
$$f : X \to \mathbb{R}$$

Single-objective Optimisation



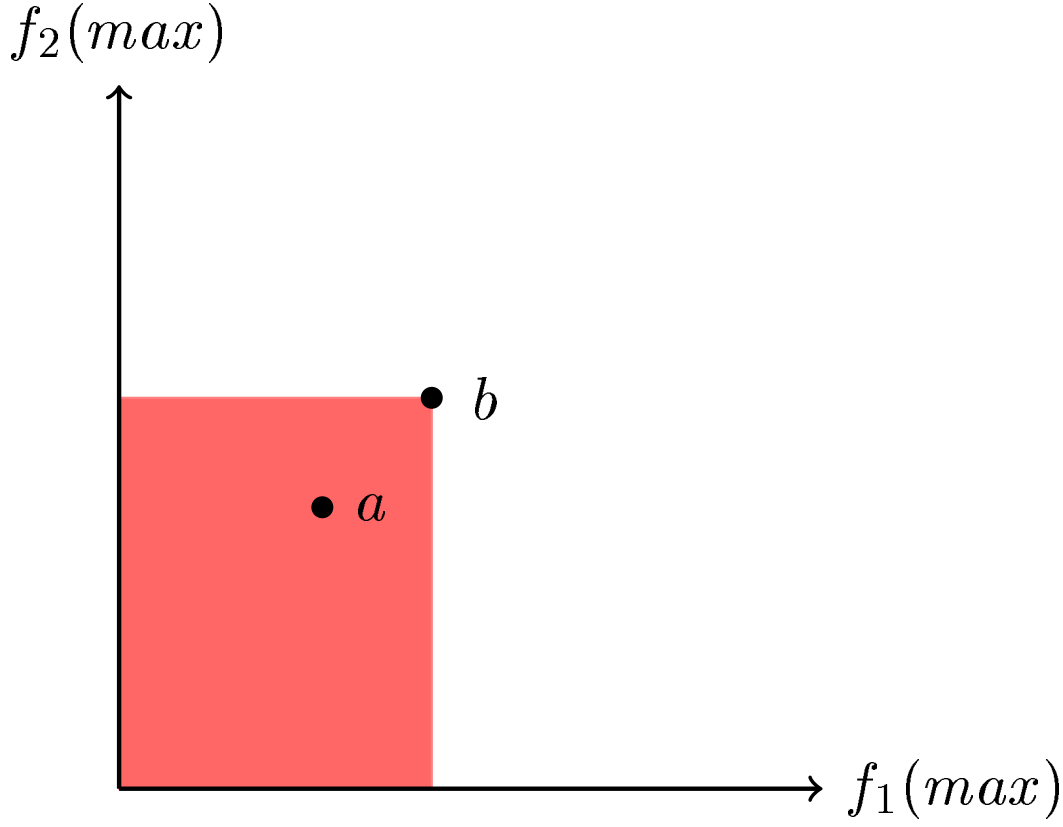
$$f : X \to \mathbb{R}$$

Multi-objective Optimisation



$$f : X \to \mathbb{R}^m, \quad m > 1$$

Pareto dominance in objective space



If $a_i \leq b_i$ for all i, then[1]

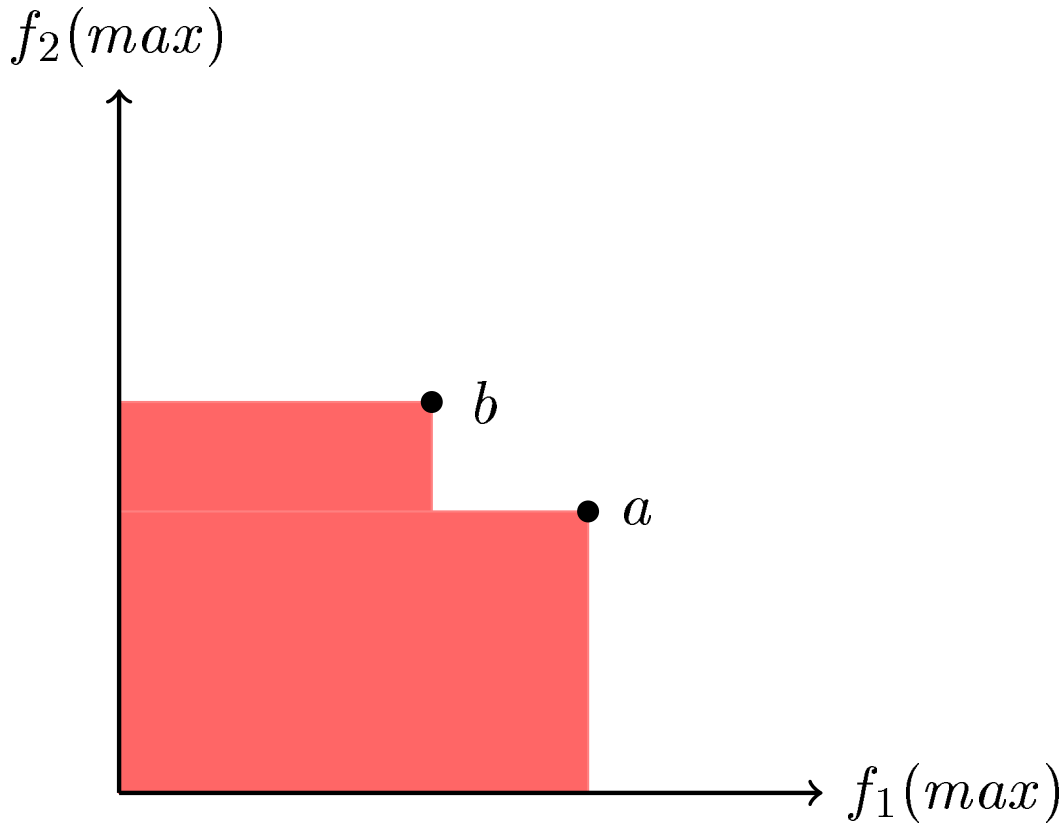
b **weakly dominates** a, denoted $a \preceq b$.

If $a \preceq b$ and there exists an i such that $a_i < b_i$, then

b **dominates** a, denoted $a \prec b$.

[1] The direction of the symbol $\preceq$ is here used differently from Deb (2001), Sect. 2.4.2, page 28, because we consider maximisation of all the objectives.

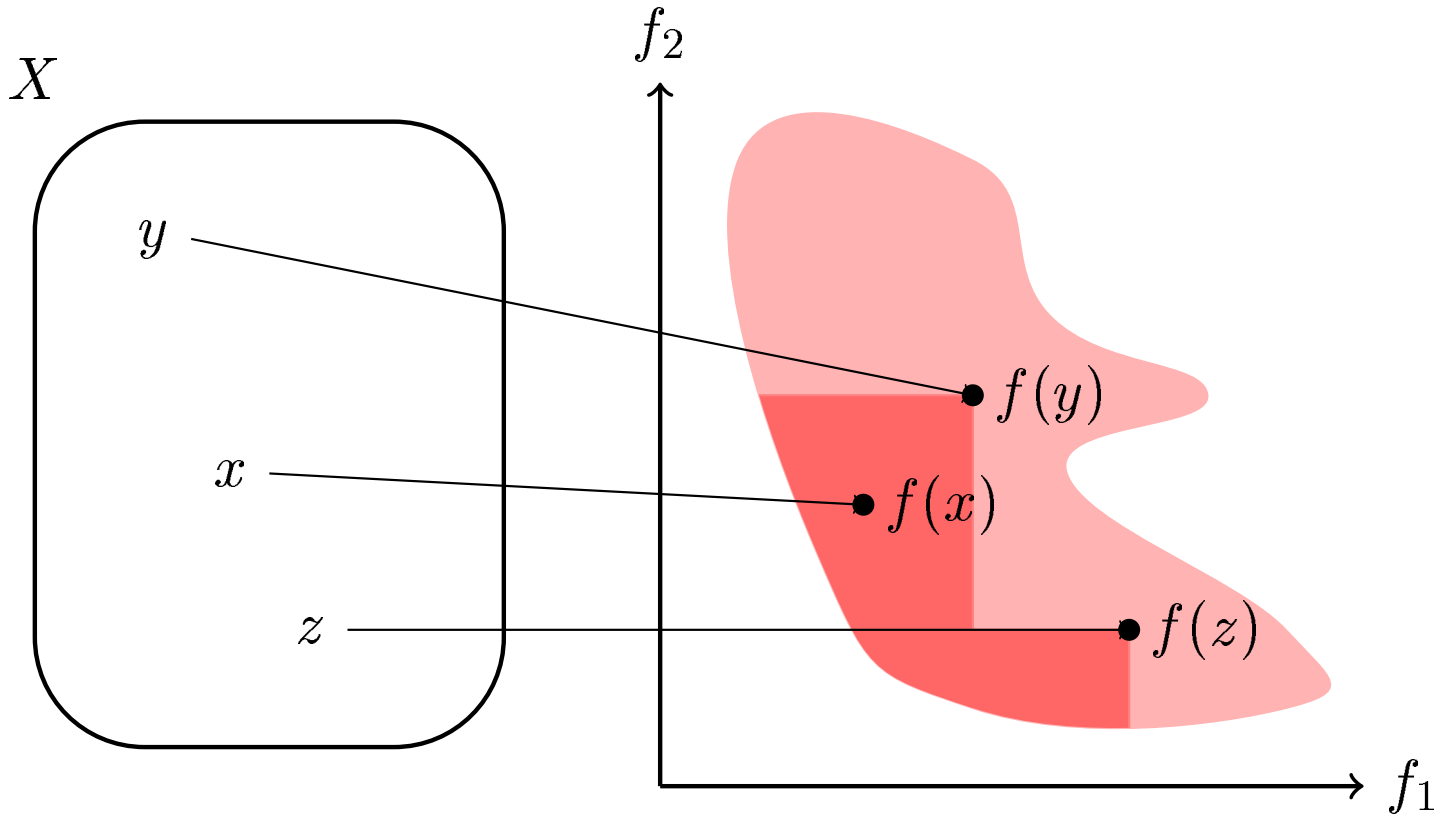
Incomparable objective vectors



If $a \not\preceq b$ and $b \not\preceq a$, then

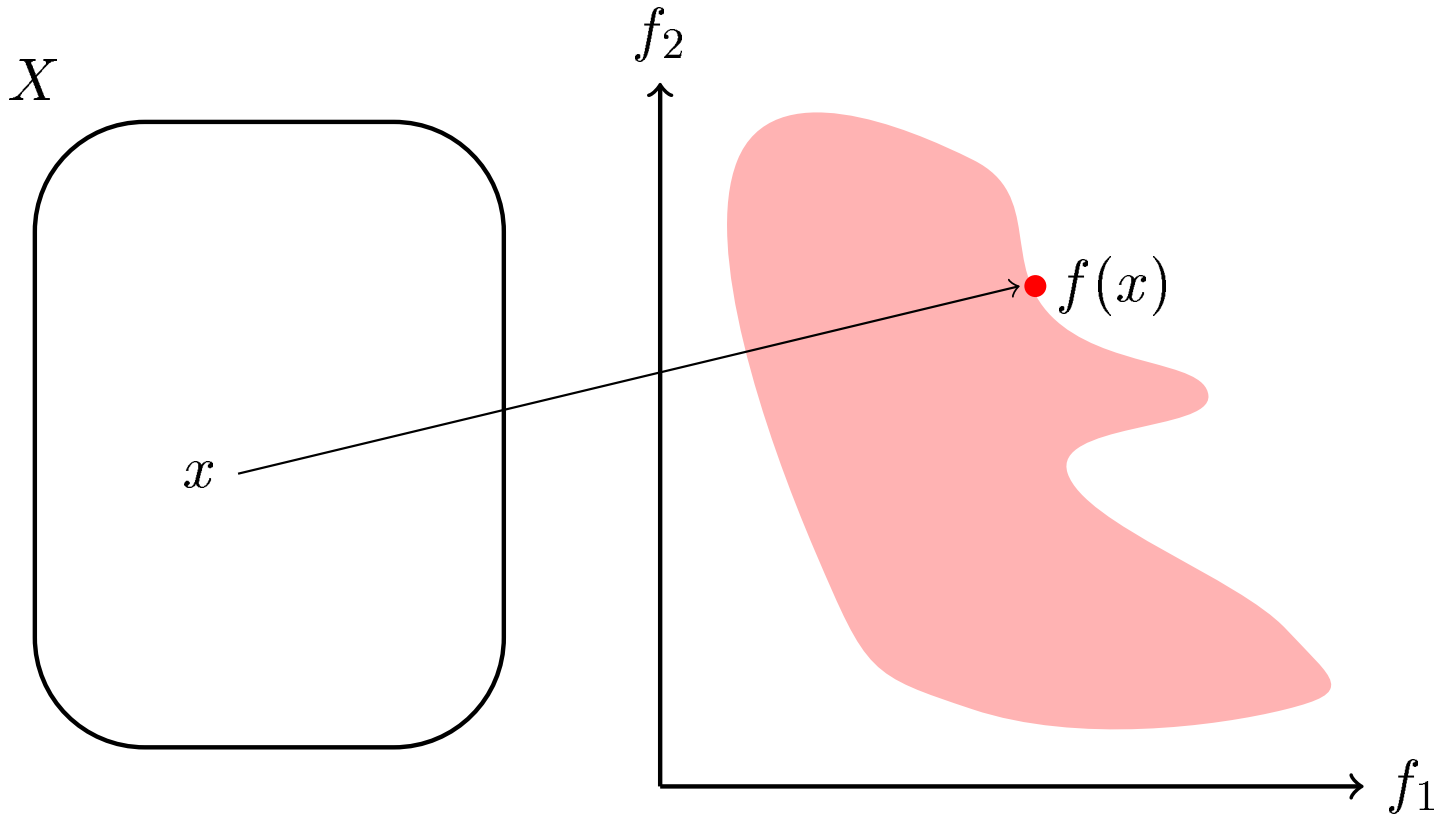
a and b are **incomparable**, denoted $a \parallel b$.

Pareto dominance in search space

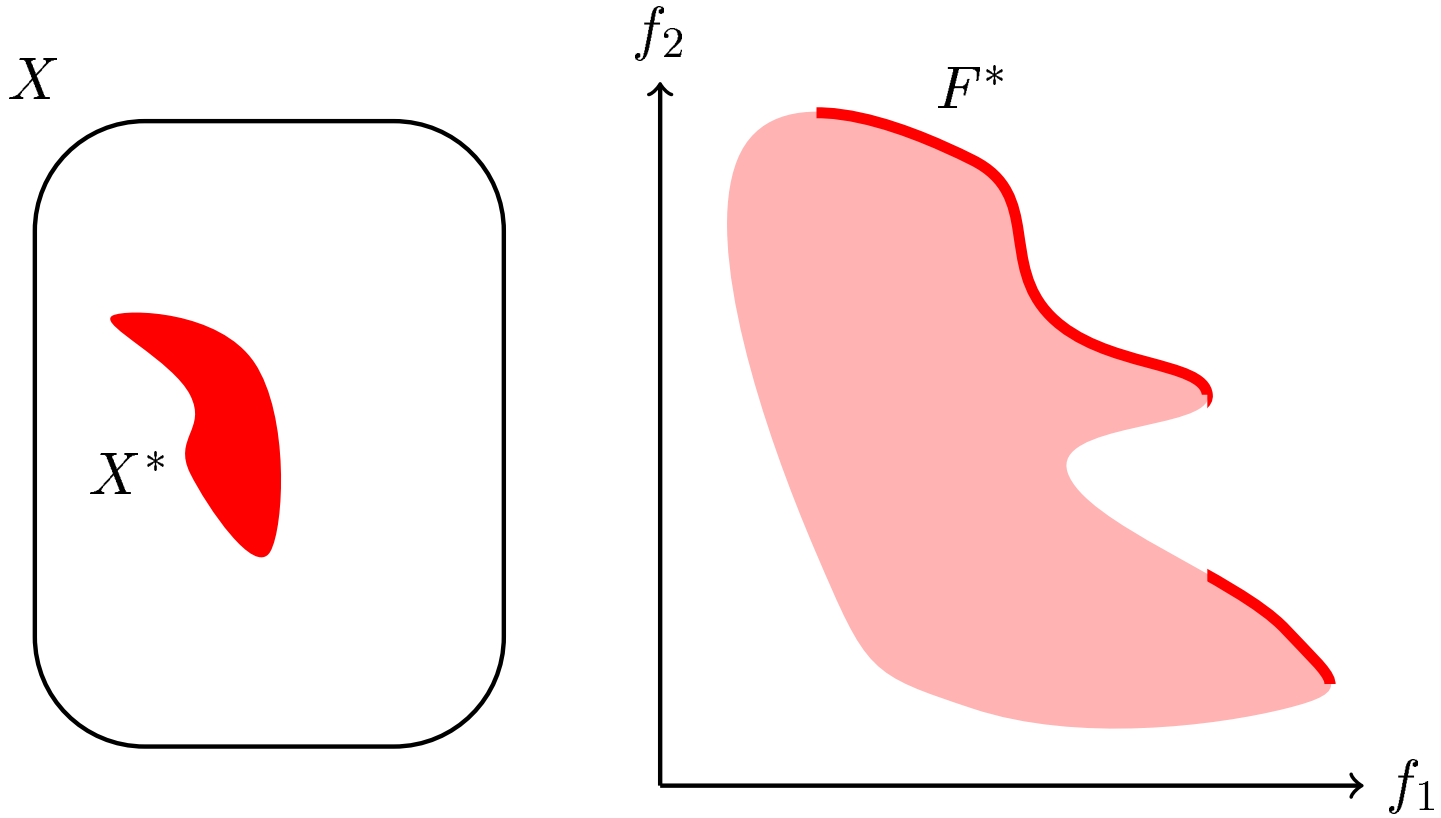


- y dominates x (x and y are comparable)
- y and z are incomparable

Pareto optimality



Pareto set and Pareto front



$$X^* := f^{-1}(F^*) \qquad F^* := \max(f(X), \preceq)$$

- A solution to a multi-objective problem f is a set of search points $Y \subset X$ that cover the Pareto front, *i.e.*, $f(Y) = F^*$.

Exercise

Given the two-objective maximisation problem

$$\textsc{LeadingOnesTrailingZeroes}(x) := (\textsc{Lo}(x), \textsc{Tz}(x)).$$

where $\textsc{Lo}$ and $\textsc{Tz}$ are functions defined over the bitstrings as

$$\textsc{Lo}(x) := \sum_{i=1}^{n} \prod_{j=1}^{i} x_j \qquad \textsc{Tz}(x) := \sum_{i=1}^{n} \prod_{j=i}^{n} (1 - x_j)$$

for example,

$$x = \overbrace{1111111111}^{\textsc{Lo}(x)}01010111\overbrace{00000000000}^{\textsc{Tz}(x)}.$$

Do the following

1. Find the Pareto set X^*.
2. Find the Pareto front F^*.
3. Draw the shape of the Pareto front.