Hypervolume Indicator Based MOEAs

Introduction to Evolutionary Computation

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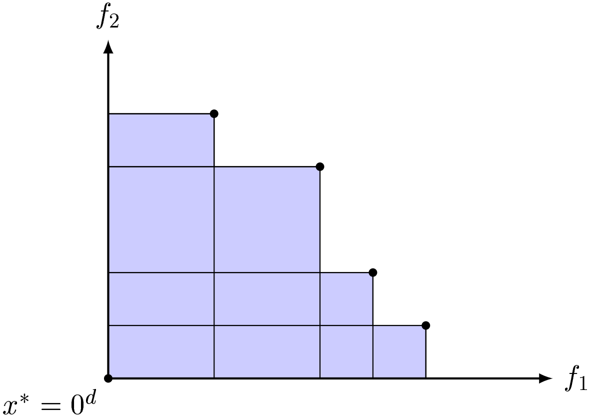
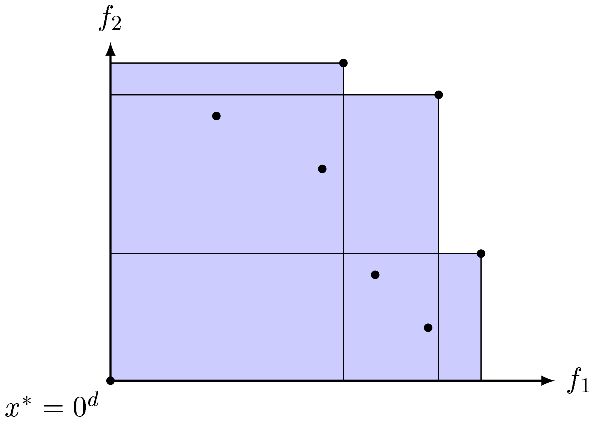
Hypervolume in Objective Space



$$\mathcal{S}(M) := \mathrm{vol}\left(\bigcup_{(x_1,\ldots,x_d)\in M} [0,x_1]\times[0,x_2]\times\cdots\times[0,x_d] \right)$$

Hypervolume in Objective Space



$$\mathcal{S}(M) := \mathsf{vol}\left(\bigcup_{(x_1,\dots,x_d)\in M} [0,x_1]\times[0,x_2]\times\cdots\times[0,x_d] \right)$$

Theoretical Properties of Hypervolume

- Hypervolume is Pareto compliant [Fleischer, 2003]

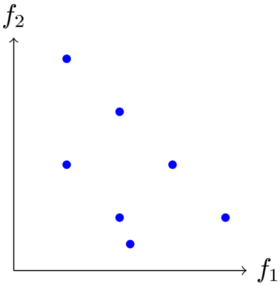
$$B \preceq A \Longrightarrow \mathcal{S}(B) \leq \mathcal{S}(A)$$

- Problem: Least Hypervolume Contributing Subset
  - Given set of n objective vectors in $\mathbb{R}^d$,
    find subset of size $\lambda < n$ that contributes least to hypervolume.
  - $O(n^{d/2} \log n + n^\lambda)$ alg [Bringmann and Friedrich, 2009].
- Calculating hypervolume $\mathcal{S}$ is $\#\mathbf{P}$-hard,
  i.e. superpolynomial runtime unless $\mathbf{P} = \mathbf{NP}$
  [Bringmann and Friedrich, 2008].

A Hypervolume based Algorithm [Beume et al., 2007]

Algorithm 1 SMS-EMOA

$P_0 \leftarrow$ init()
$t \leftarrow 0$
repeat
    $q_{t+1} \leftarrow$ generate(P_t)
    $P_{t+1} \leftarrow$ reduce($P_t \cup \{q_{t+1}\}$)
    $t \leftarrow t + 1$
until termination condition met.



Algorithm 2 reduce(Q)

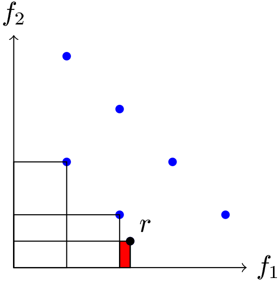
$\{R_1, R_2, ..., R_v\} \leftarrow$ fast-nondominated-sort(Q)
$r \leftarrow \arg\min_{s \in R_v} \Delta_{\mathcal{S}}(s, R_v)$
return $Q \setminus \{r\}$

$$\Delta_{\mathcal{S}}(s, R_v) := \mathcal{S}(R_v) - \mathcal{S}(R_v \setminus \{s\}).$$

A Hypervolume based Algorithm [Beume et al., 2007]

Algorithm 3 SMS-EMOA

```
P_0 ← init()
t ← 0
repeat
    q_{t+1} ← generate(P_t)
    P_{t+1} ← reduce(P_t ∪ {q_{t+1}})
    t ← t + 1
until termination condition met.
```



Algorithm 4 reduce(Q)

```
{R_1, R_2, ..., R_v} ← fast-nondominated-sort(Q)
r ← argmin_{s∈R_v} Δ_S(s, R_v)
return Q \ {r}
```

$$\Delta_{\mathcal{S}}(s, R_v) := \mathcal{S}(R_v) - \mathcal{S}(R_v \setminus \{s\}).$$

SMS-EMOA Empirical Results

1662 N. Beume et al. / European Journal of Operational Research 181 (2007) 1653–1669

Table 2
SMS-EMOA and best results from [9] of other EMOA

	Algorithm	Convergence measure		$\mathcal{S}$ metric	
		Average	Standard deviation	Average	Standard deviation
DTLZ1	SPEA2	0.0033377	3.54e−02	0.315981	6.98e−04
	ϵ-MOEA	**0.00245**	9.52e−05	0.298487	NC
	SMS-EMOA	0.0029175	1.72e−04	0.316930	5.30e−05
	SMS-EMOA dp	0.0028909	1.06e−04	**0.316936**	8.38e−05
DTLZ2	C-NSGA-II	0.00986	8.8e−04	NC	NC
	SMS-EMOA	**0.0063652**	3.20e−04	0.757911	4.49e−05
	SMS-EMOA dp	0.0065383	5.12e−04	**0.757994**	4.74e−05
DTLZ3	ϵ-MOEA	0.0122290	2.23e−03	NC	NC
	SMS-EMOA	0.0071626	5.94e−04	0.755294	2.22e−03
	SMS-EMOA dp	**0.0069858**	4.28e−04	**0.755443**	7.9e−04
DTLZ4	ϵ-MOEA (6)	0.0097755	2.0e−04	NC	NC
	SMS-EMOA (4)	**0.0065006**	3.39e−04	0.757949	8.65e−05
	SMS-EMOA dp (5)	0.0065193	4.42e−04	**0.757967**	3.87e−05

References I

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