Problem 1

When
\[ \exists \alpha, \beta, (\text{lseg } \alpha (i, j) \ast \text{lseg } \beta (j, k)) \land \gamma = \alpha \cdot \beta, \]
we say that \( j \) is an *interior pointer* of the list segment described by \( \text{lseg } \gamma (i, k) \).

1. Give an assertion describing a list segment with two interior pointers \( j_1 \) and \( j_2 \), such that \( j_1 \) comes before than, or at the same point as, \( j_2 \) in the ordering of the elements of the list segment.

2. Give an assertion describing a list segment with two interior pointers \( j_1 \) and \( j_2 \), where there is no constraint on the relative positions of \( j_1 \) and \( j_2 \).

3. Prove that the first assertion implies the second.

Problem 2

A *braced list segment* is a list segment with an interior pointer \( j \) to its last element; in the special case where the list segment is empty, \( j \) is \text{nil}. Formally,

\[ \text{brlseg } \epsilon (i, j, k) \overset{\text{def}}{=} \text{emp } \land i = k \land j = \text{nil} \]

\[ \text{brlseg } \alpha \cdot a (i, j, k) \overset{\text{def}}{=} \text{lseg } \alpha (i, j) \ast j \mapsto a, k. \]

1. Prove the assertion

\[ \text{brlseg } \alpha (i, j, k) \Rightarrow \text{lseg } \alpha (i, k). \]

2. Give an annotated specification of a command that changes the final pointer of a braced list segment from \( k \) to \( l \). The command should not construct or deallocate any list structure, and it should not change the value of \( j \). It should satisfy

\[ \{ \text{brlseg } \alpha (i, j, k) \} \cdots \{ \text{brlseg } \alpha (i, j, l) \}. \]
3. Give an annotated specification of a command that concatenates two braced list segments. The command should not construct or deallocate any list structure, and it should not change the value of \( k_2 \). It should satisfy

\[
\{ \text{brlseg } \alpha (i_1, j_1, k_1) \star \text{brlseg } \beta (i_2, j_2, k_2) \}\]

\[
\cdots
\]

\[
\{ \text{brlseg } \alpha \cdot \beta (i_1, j, k_2) \}.
\]

4. Give an annotated specification of a command that appends an item \( a \) at the beginning of a braced list segment. The command should not change the value of \( k \), and should satisfy

\[
\{ \text{brlseg } \alpha (i, j, k) \} \cdots \{ \text{brlseg } a \cdot \alpha (i, j, k) \}.
\]

5. Give an annotated specification of a command that appends an item \( a \) at the end of a braced list segment. The command should not change the value of \( k \), and should satisfy

\[
\{ \text{brlseg } \alpha (i, j, k) \} \cdots \{ \text{brlseg } a \cdot \alpha (i, j, k) \}.
\]

6. Give an annotated specification of a command that deletes an item \( a \) from the beginning of a nonempty braced list segment. The command should not change the value of \( k \), and should satisfy

\[
\{ \text{brlseg } a \cdot \alpha (i, j, k) \} \cdots \{ \text{brlseg } \alpha (i, j, k) \}.
\]