

# Exercise Sheet 0: Solutions

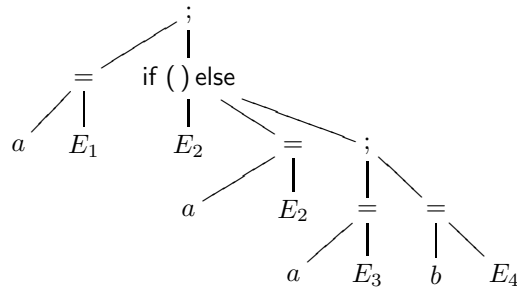
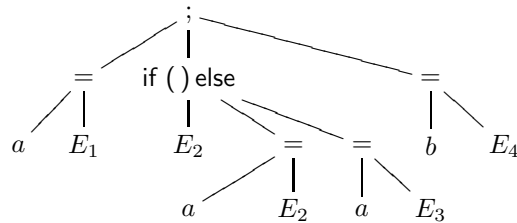
January 27, 2010

## Exercise 1: Inductive definitions

(a)

- If  $X$  is a variable, then  $X$  is an expression.
- If  $X$  is a variable and  $E$  is an expression, then  $X = E$  is a command.
- If  $C_1, \dots, C_n$  are commands then  $C_1; \dots; C_n$  is a command.
- If  $E$  is an expression and  $C_1$  and  $C_2$  are commands, then  $\text{if } (E) C \text{ else } C$  is a command.
- If  $E$  is an expression and  $C$  is a command, then  $\text{while } (E) C$  is a command.

(b)



(c)  $a = E_1; \text{if } (E_2) a = E_2 \text{ else } \{a = E_3; b = E_4\}$

This matches the second syntax tree in part (b).

## Exercise 2: Production rules

$$\begin{aligned}
 E & ::= X \\
 C & ::= X = E \\
 C & ::= C_1; \dots; C_n \\
 C & ::= \text{if } (E) C_1 \text{ else } C_2 \\
 C & ::= \text{while } (E) C_1
 \end{aligned}$$

## Exercise 3: Inference rules

(a)

$$\begin{array}{c}
 \frac{X \text{ Variable}}{X \text{ Expression}} \qquad \frac{X \text{ Variable} \quad E \text{ Expression}}{X = E \text{ Command}} \\
 \frac{C_1 \text{ Command} \quad \dots \quad C_n \text{ Command}}{C_1; \dots; C_n \text{ Command}} \\
 \frac{E \text{ Expression} \quad C_1 \text{ Command} \quad C_2 \text{ Command}}{\text{if } (E) C \text{ else } C \text{ Command}} \qquad \frac{E \text{ Expression} \quad C \text{ Command}}{\text{while } (E) C \text{ Command}}
 \end{array}$$

(b) We abbreviate Variable, Command and Expression.

$$\frac{\frac{a \text{ Var} \quad E_1 \text{ Expr}}{a = E_1 \text{ Com}} \quad \frac{\frac{E_2 \text{ Expr} \quad \frac{a \text{ Var} \quad E_2 \text{ Expr}}{a = E_2 \text{ Com}}}{\text{if } (E_2) a = E_2 \text{ else } \{a = E_3; b = E_4\} \text{ Com}} \quad \frac{\frac{a \text{ Var} \quad E_3 \text{ Expr}}{a = E_3 \text{ Com}} \quad \frac{b \text{ Var} \quad E_4 \text{ Expr}}{b = E_4 \text{ Com}}}{a = E_3; b = E_4 \text{ Com}}}{a = E_1; \text{if } (E_2) a = E_2 \text{ else } \{a = E_3; b = E_4\} \text{ Com}}$$