

Exercise Sheet 1

Attempt as many questions as you can during the exercise class, and work on the remainder at home. Hand in your solutions to the tutor in the Friday exercise class, on 27 Jan, 2010.

Exercise 1: Introduction to the lambda calculus

- a. In what ways does Java's way of defining functions (i.e. methods) differ from that of the λ -calculus?
- b. Translate the following Java expressions into λ -calculus notation:
 - i. `sin(x+3)`
 - ii. `length(y)+z`
 - iii.

```
public static int quot(double x, double n)
    { return (int)(x/n); }
```

Exercise 2: Parsing lambda terms

Draw the syntax trees for the following λ -terms:

- a. $\lambda x. \lambda y. x$
- b. $\lambda x. \lambda y. \lambda z. xyz$
- c. $(\lambda x. xx)(\lambda x. xx)$

Exercise 3: beta reduction

- a. Consider the λ -term $(\lambda f. \lambda x. f(fx)) (\lambda y. * y 2) 5$.
 - i. Draw its abstract syntax tree.
 - ii. β -reduce the term as far as possible.
- b. Let S be the term $\lambda x. \lambda y. \lambda z. (xz)(yz)$ and K the term $\lambda x. \lambda y. x$. Reduce SKK to normal form. (Hint: This can be messy if you are not careful. Keep the abbreviations S and K around as long as you can and replace them with their corresponding λ -terms only if you need to. It becomes very easy then.)
- c. Consider the following lambda terms:

$$\begin{aligned} M &= \lambda x. ((\lambda z. z x)((\lambda r. \lambda s. s r) y f)) \\ N &= \lambda x. ((\lambda z. \lambda u. \lambda v. u v z)x f y) \end{aligned}$$

Use β -reduction to show that M and N are β -equivalent.