

## *Exercise Sheet 7*

Attempt as many questions as you can during the exercise class, and work on the remainder at home. Hand in your solutions to the tutor in the Friday exercise class, on 26 March, 2010.

In this exercise, we want you to evaluate terms using binding environments, each environment being written as a linked list frames, using this notation:

$$E = F_0 \leftarrow F_1 \leftarrow \dots$$

Each frame must be a set of variable bindings such as  $\{x_1 \mapsto V_1, \dots, x_n \mapsto V_n\}$ .

Lambda abstraction terms evaluate to closures, written in the form  $\text{cls}(\lambda x. M, E)$  where  $E$  is the environment that is in effect for the lambda abstraction.

### Exercise 1: Environment frames

- a. Evaluate the expression

$$((\lambda x. x + y) 100)$$

starting from an global environment consisting of just one frame,  $F_0 = \{y \mapsto 4\}$ .

- b. Evaluate the expression

$$(\lambda f. \lambda y. f y) (\lambda x. x + y) 100$$

starting from the same environment with  $F_0$ .

- c. Draw a diagram in the style of Abelson and Sussman, showing all the environment frames and static links. Label the frames with the procedures whose activations (calls) created them.

### Exercise 2: Environment frames, again

Recall the *times* function:

$$\text{times} = (\lambda n. \lambda f. \text{if } (n = 0) (\lambda x. x) \\ (\lambda x. \text{times } (n - 1) f (f x)))$$

Assume that it is defined at the top level and that the global frame is

$$F_0 = \{\text{times} \mapsto \text{cls}(\lambda n. \lambda f. \dots, F_0), x \mapsto 20\}$$

Evaluate the term

$$\text{let } inc = \lambda z. z + 1 \\ \text{in } \text{times } 3 \text{ inc } x$$

using the environment model. Show all the environments and frames that arise during the evaluation.

### Exercise 3: SECD Machine

- a. Show the evaluation of the expression  $((\lambda x. x) 2)$  using the SECD machine. The initial configuration is:

$$([], \{\}, [(\lambda x. x) 2], [])$$

At each step specify the transition rule being employed. (You can make a table as shown in the part c below.)

- b. Show that there is a multiple step transition sequence of the following form:

$$\text{DBL-APPLICATION } (S, E, [(M N_1 N_2) | C], D) \longrightarrow^* (S, E, [M, N_1, \text{app}, N_2, \text{app} | C], D)$$

- c. Shown below are the first few steps of the evaluation for the expression  $((\lambda f. \lambda y. f y) (\lambda x. + x y) 1)$ . Mention the transition rules used for each step and verify that the rules have been applied correctly.

|    | <u>stack</u>  | <u>environment</u> | <u>control</u>   | <u>dump</u> |
|----|---|--------------------|--|-------------|
|    |   | $E_1$              | $((\lambda f. \lambda y. f y) (\lambda x. + x y) 1)$                         | $[]$        |
| 1. |   | $E_1$              | $((\lambda f. \lambda y. f y) (\lambda x. + x y)), 1, \text{app}$            | $[]$        |
| 2. |   | $E_1$              | $(\lambda f. \lambda y. f y), (\lambda x. + x y), \text{app}, 1, \text{app}$ | $[]$        |
| 3. | $\text{cls}(\lambda f. \lambda y. f y, E_1)$                                    | $E_1$              | $(\lambda x. + x y), \text{app}, 1, \text{app}$                              | $[]$        |
| 4. | $\text{cls}(\lambda f. \lambda y. f y, E_1), \text{cls}(\lambda x. + x y, E_1)$ | $E_1$              | $\text{app}, 1, \text{app}$  | $[]$        |
| 5. |   | $E_2$              | $\lambda y. f y, \text{rtn}, 1, \text{app}$                                  | $[E_1]$     |
| 6. | $\text{cls}(\lambda y. f y, E_2)$   | $E_2$              | $\text{rtn}, 1, \text{app}$  | $[E_1]$     |
| 7. | $\text{cls}(\lambda y. f y, E_2)$   | $E_1$              | $1, \text{app}$  | $[]$        |
| 8. | $\text{cls}(\lambda y. f y, E_2), 1$  | $E_1$              | $\text{app}$   | $[]$        |
| 9. |   | $E_3$              | $f y, \text{rtn}$  | $[E_1]$     |

where the environments mentioned are:

$$\begin{aligned}
 E_1 &= \{y \mapsto 4, z \mapsto 5\} \\
 E_2 &= \{y \mapsto 4, z \mapsto 5, f \mapsto \text{cls}(\lambda x. + x y, E_1)\} \\
 E_3 &= \{y \mapsto 1, z \mapsto 5, f \mapsto \text{cls}(\lambda x. + x y, E_1)\}
 \end{aligned}$$

- d. Continue the evaluation of part (c). You should proceed step by step until you obtain  $+ x y$  in the control sequence. After that, you can guess the remaining steps without elaborating all the detail.

#### Exercise 4: [Optional question] SECD transition rules

Recall that the expression form **let**  $x = M$  **in**  $N$  can be desugared into lambda calculus. Define a transition rule for the SECD machine that allows it to directly reduce **let** expressions without translating it into lambda calculus.