

## Exercise Sheet 5

Attempt as many questions as you can during the exercise class, and work on the remainder at home. Hand in your solutions to the tutor in the Friday exercise class, on 26 Feb, 2010.

### Exercise 1: Type checking and typing derivations

- a. Type check the following terms in the applied lambda calculus, and derive their types.
  - i.  $\lambda x:\mathbf{int}. \lambda y:\mathbf{bool}. x$ .
  - ii.  $\lambda p:\mathbf{int} \rightarrow \mathbf{bool}. \lambda f:\mathbf{int} \rightarrow \mathbf{int}. \lambda x:\mathbf{int}. p(f\ x)$
- b. Write their typing derivations (either in tree form or linear form).

### Exercise 2: More type checking

Type check the following term and derive its type:

$$\mathbf{Y} (\lambda f : \mathbf{int} \rightarrow \mathbf{bool}. \lambda n : \mathbf{int}. \mathbf{if}_{\mathbf{bool}} (n = 0) \mathbf{true} (f (n - 1)))$$

### Exercise 3: Type rules

The typed constructs  $\mathbf{let} x : T = M \mathbf{in} N$  and  $\mathbf{letrec} x : T = M \mathbf{in} N$  can be regarded as syntactic sugar for the basic typed lambda calculus.

- a. Give appropriate desugaring translations of these constructs into the typed lambda calculus.
- b. Formulate type rules for the new constructs.
- c. Verify that desugaring any type-correct  $\mathbf{let}$  or  $\mathbf{letrec}$  term gives a type-correct term in the typed lambda calculus.

### Exercise 4: Type inference

Calculate the principal types for the following terms:

- a.  $\lambda x. \lambda y. y\ x$ .
- b.  $\lambda x. \lambda y. \lambda z. (x\ z) (y\ z)$ .
- c.  $\mathbf{Y} (\lambda f. \lambda x. \mathbf{if} (\mathbf{null}\ x) 0 (1 + f(\mathbf{cdr}\ x)))$