

# Fun with Constructive Modalities

Valeria de Paiva

Cuil, Inc.

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# An applied logician's job is never done...

- When modeling an implemented system as a logic you can start from the system



- Or you can start from logics that could fit it
- Hopefully the two meet up...

# This talk: Off-the-shelf logical systems



## The pieces of the puzzle...

- FOL
- HOL
- Modal logic
- Description logic
- Hybrid logic
- MCS/LMS
- Intensional Logic
- Etc...

# Outline

- Motivation
- Brief history?
- Constructive modal logic
- Constructive hybrid logic
- Constructive description logic
- Discussion

# Motivation: Constructive Modalities?

- Modalities and modal logic: the **most** successful logical framework in CS
- Temporal logic, knowledge operators, BDI models, security issues, AI, natural language understanding and inference, databases, etc..
- Logic used both to create logical representation of information and to reason about it
- Usually **classical** modalities
- What about **constructive** modalities?

# Motivation: Constructive Modalities?

- **Constructive** logic: a logical basis for programming via Curry-Howard correspondences
- Modalities extremely useful
- Constructive modalities twice as useful?
- examples from applications abound
- **Which** constructive modalities?
- Usual phenomenon: classical facts can be construed in many different ways constructively, choosing is an art...

# • **Constructive reasoning: what, why, how...**

- What: Reasoning principles that are safer
- if I ask you whether “There is  $x$  such that  $P(x)$ ”,
- I'm happier with an answer “yes,  $x_0$ ”, than with an answer “yes, for all  $x$  it is not the case that not  $P(x)$ ”.
- Why: want reasoning to be as precise and safe as possible
- How: constructive reasoning as much as possible, but classical if need be

# A brief history

- Debates over constructive or classical logics since the beginning of the 20<sup>th</sup> century
- Modal logics from 1920's - Lewis
- Kripke-like semantics in the 60s.
- Connections constructive/modal logic:
  - Algebraic McKinsey/Tarski 30s
  - Kripke semantics, for both 65
  - Modal type theories, 90's
- Putting constructive and modal together:
- Fitch 1948 MIPC, Bull 1966, Prawitz 1965, Curry,



# More brief history...

- Intuitionistic modal logic:
  - Analogy
  - Semantics
  - Translations
  - Other
- Fisher-Servi 80's,
- Bozic-Dosen, 84, Volter/Zacharyashev 88
- Simpson, Gabbay, Masini/Martini early 90's
- Mendler, Fairtlough, Bierman/dePaiva, etc
- Overviews: Goldblatt, IMLA'04 (dePaiva, Mendler, Gore')

# Constructive modal logics

- Basic ideas:
  - Box, Diamond like forall/exists
  - Intuitionistic logic like S4-modal logic,
  - where  $A \multimap B = \Box A \rightarrow B$
  - Combining modalities not that easy...
- To have “intuitionistic modal logic” need to have two modalities, how do they interact?
  - Commuting squares possibilities
- Adding syntax: hypersequents, labelled deduction systems, adding semantics to syntax (many ways...)

# Constructive modal logics

- Personal programme:
- constructive modal logics with axioms, sequents and natural deduction formulations
- Also with algebraic, Kripke and categorical semantics
- With translations between formulations and proved equivalences/embeddings
- Translating proofs more than simply theorems
- broad view of constructive and/or modality

# Simpson Desiderata for IML '94

- IML is a conservative extension of IPL.
- IML contains all substitutions instances of theorems of IPL and is closed under modus ponens.
- Adding excluded middle to IML yields a standard classical modal logic
- If “A or B” is a theorem of IML either A is a theorem or B is a theorem too.
- Box and Diamond are independent in IML.
- (Intuitionistic) Meaning of the modalities, wrt it IML is sound and complete

# Extensions: Description and Hybrid Logics

- Description and Hybrid logics are closely associated with modal logics
- Both classes tend to be classical logics, for the same reasons above
- We discuss both constructive hybrid logics (Brauner/dePaiva 03) and constructive description logics (dePaiva05) in turn.

# Constructive Hybrid Logic?

- What are hybrid logics?
- Extension of modal logic, where we make part of the syntax of the formulae the worlds at which they're evaluated.
- Add to basic modal logic second kind of propositional symbols (nominals) and satisfaction operators
- A nominal is assumed to be true at exactly one world
- A formula like  $a:A$  where  $a$  is a nominal and  $A$  is a formula is called a satisfaction statement

# Constructive Hybrid Logic!

- Brauner/dePaiva ('03, '05)
- Which kind of constructive?
- Depends on kind of constructive modal logic
- Many choices for syntax and for models.
- Our choice: modal base Simpson-style, Natural Deduction style.
- Results: IHL as a ND system, models, soundness and completeness, extensions to geometric theories
- Open problem: hybrid system CK style?...

# What Are Description Logics?

- A family of logic based Knowledge Representation formalisms
  - Descendants of **semantic networks** and **KL-ONE**
  - Describe domain in terms of **concepts** (classes), **roles** (properties, relationships) and **individuals**
- Distinguished by:
  - **Formal semantics** (typically model theoretic)
    - » Decidable fragments of FOL (often contained in  $C_2$ )
    - » **Closely related to Propositional Modal, Hybrid & Dynamic Logics**
    - » Closely related to Guarded Fragment
  - Provision of **inference services**
    - » Decision procedures for key problems (satisfiability, subsumption, etc)
    - » Implemented systems (highly optimised)

Thanks Ian Horrocks!

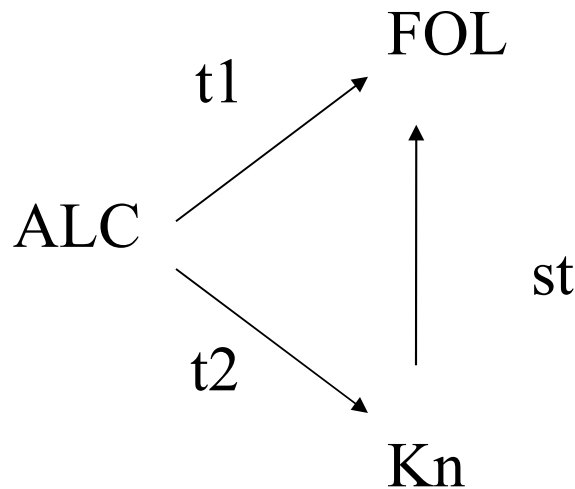


# DL Basics

- **Concepts (formulae/unary predicates)**
  - E.g., Person, Doctor, HappyParent, etc.
- **Roles (modalities/relations)**
  - E.g., hasChild, loves
- **Individuals (nominals/constants)**
  - E.g., John, Mary, Italy
- **Operators (for forming concepts and roles) restricted so that:**
  - Satisfiability/subsumption is decidable and, *if possible*, of low complexity
  - No need for explicit use of variables
  - Features such as counting (**graded modalities**) succinctly expressed

# What are description logics?

- A sublogic of FOL?
- Or a sublogic of Modal logic?



# Constructive Description Logic via Translation

- DL can be defined via t1 translation into FOL
- To constructivize it transform FOL into IFOL  
Call system IALC
- DL can be defined via t2 translation into multimodal K (Schildt91)
- Need to choose a constructive K
- Using IK (Simpson) call system iALC, using CK (Mendler & de Paiva) call system cALC

# Two translations

- Into first-order logic  $t1:ALC \rightarrow FOL$
- concept  $C$  maps to  $C(x)$ , role  $R$  maps to relation, quantifiers the point
- Into modal logic  $t2:ALC \rightarrow Kn$ , roles into boxes, diamonds

# Constructive Description Logic: IALC

- Basic idea: translate description syntax using  $t_1$  into IFOL, instead of FOL
- No excluded middle, no duality between existential and universal quantifiers, no duality between conjunction and disjunction
- Pros: IFOL fairly standard
  - Can provide IALC models easily
- Cons: semantics of IFOL more complicated...
- Result: Given IALC model  $M$ , given formula  $A$   $M$  satisfies  $A$  iff  $M$  satisfies  $t_1(A)$ , that is  $t_1$  is truth-preserving translation

# Constructive Description Logic: iALC and cALC

- Basic idea: translate description syntax using  $\mathcal{L}_2$  into constructive modal logic, instead of classical modal  $\mathcal{K}_n$ .
- Which constructive  $\mathcal{K}$ ?
- If Simpson's  $\mathcal{IK} \rightarrow \text{iALC}$ ,
- if Mendler/de Paiva  $\mathcal{CK} \rightarrow \text{cALC}$
- Difference: distribution of possibility over disjunction and nullary one:
  - $\text{Dia}(A \text{ or } B) \rightarrow \text{Dia } A \text{ or } \text{Dia } B$
  - $\text{Dia}(\text{false}) \rightarrow \text{false}$

# Constructive Description Logic II: iALC

- Note that translation  $t_2$  into constructive modal logic is the same for both iALC and cALC, just the target language change.
- For iALC, can use our work on intuitionistic hybrid logic
- Models easily described
- Framework: several modal logics + geometric theories
- Referee's remark: complexity?

# Constructive Description Logic II: cALC

- For cALC, can use our work on an extended Curry-Howard isomorphism for constructive modal logic
- No Framework: can only do S4 and K
- Can do Kripke models and categorical models
- Haven't investigated interpolation, decidability or complexity
- New work by Mendler and Schiele



# Related Work

- Mandler/Schiele on constructive description for auditing
- Bozzato, Ferrari et al, CHI for ALC, diff HH and A Rademaker?
- Odintsov and Wansing's "Inconsistent-tolerant description logic I and II"
  - Motivation is paraconsistency, not constructivity
- Hofmann's "Proof theoretical Approach to DL"
  - Motivation fixpoints in description logics and their complexity
- Straccia's and Patel-Schneider's papers on 4-valued description logic
  - Motivations are fuzziness and uncertainty

# Discussion

- This is very preliminary
- While it is true that constructive reasoning multiply concepts, there should be criteria to identify best system(s?)
- Part of bigger programme of constructivizing logics for computer science
- Want to keep criteria both from theory and applications
- Next steps: criteria from modal/hybrid logic, bisimulations, complexity bounds, temporal logics, etc...

# References

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# Thanks!

papers/preprints at

<http://www.cs.bham.ac.uk/~vdp/publications/papers.html>