CONSTRUCTIVE CK FOR CONTEXTS

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This note describes possible world semantics for a constructive modal logic CK. The system CK is weaker than other constructive modal logics K as it does *not* satisfy distribution of possibility over disjunctions, neither binary ($\Diamond(A \vee B) \to \Diamond A \vee \Diamond B$) nor nullary ($\Diamond\bot \to \bot$). Our intuition is that $\Diamond\bot$ should not constructively imply $\bot$, thus we dispense with both forms of distribution. We are interested in this version of constructive K for its application as contexts in AI [3]. However, logicians interested in contexts prefer their semantics possible worlds style, while system CK first had only a categorical semantics [2]. Wijesekera [4] investigated possible-worlds semantics of a system similar to CK, without the binary distribution, but satisfying the nullary one. We had hoped that an adaptation of Wisejekera's results would work for us, but a flaw in his proofs meant that his results had to be reworked. We also wanted a possible-worlds semantics for CK because it can be seen as a natural generalization of our work [1] on a constructive and categorical version of modal S4.

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