Lens System Design and Re-Engineering with Evolutionary Algorithms

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Abstract

This paper presents some lens system design and re-engineering experimentations with genetic algorithms and genetic programming. These Evolutionary Algorithms (EA) were successfully applied to a design problem that was previously presented to expert participants of an international lens design conference. Comparative results demonstrate that the use of EA for lens system design is very much human-competitive.

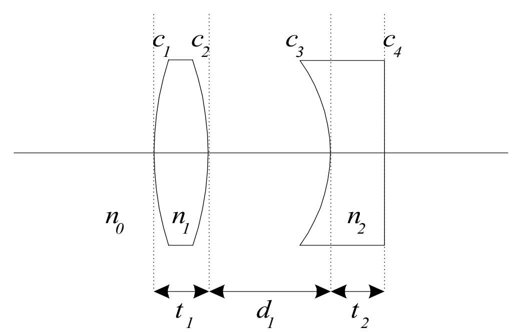

Figure 1: Parameters of a Two Lenses System. The n_i variables denote the refractive indexes of the media, the c_i represent the lens surface curvatures, the t_i are the lens thicknesses, and d_1 is the lens spacing.

1 INTRODUCTION

Designing a lens system is a complex task currently done by experimented optical engineers, using CAD tools that can optimize a roughly shaped design. The work presented in this paper is motivated by a desire to completely automate this design task, using Genetic Algorithms (GA) and Genetic Programming (GP) techniques.

The paper first presents the theory related to lens systems, before addressing a brief survey of modern design methods. Then, experimental results are presented for the automatic design of a benchmark problem for both the design (using GA and GP) and the re-engineering (using GP) of a lens system.

2 THEORY ON LENS SYSTEM DESIGN

A lens system is an arrangement of lenses with different refractive indexes, surface curvatures, and thicknesses. Figure 1 shows an example of a 2 lenses system. Given an object of a certain size, at a certain distance, its function is to produce an image of this object. Although many lens arrangements can generate images of the same size, the problem of lens system design is to seek the one with the least amount of aberration.

Aberrations are the difference between a real image and the corresponding approximated image computed with Gauss optics (O'Shea, 1985). Gauss optics constitute a usable framework to characterize an optical system with various Gaussian constants such as the effective focal length, stop, f-number of the system, and image distance and magnification. Aberrations come from the fact that Gauss optics are used during the design process; real physics of lens systems are too complex to be usable.

To characterize lens systems we need to do what is called ray tracing. Starting at a given point on the object and a given initial angle, a ray trace is the computation of the trajectory of a light ray through the

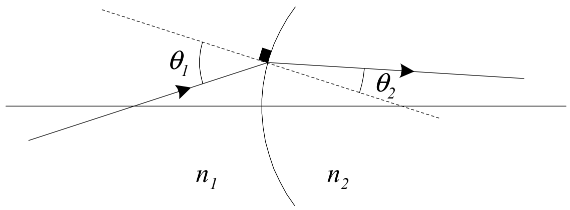

Figure 2: Illustration of Snell-Descartes First Law of Refraction

optical system until it reaches the image plane. The exact (real) ray trace is obtained from the first law of refraction (Snell-Descartes) that governs the behavior of light passing through the interface between two media having different refractive indexes. The path of a ray passing from medium 1 to medium 2 obeys the following equation:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

where n_1 and n_2 are refractive indexes of media 1 and 2, and θ_1 and θ_2 are incident and refracted angles relative to the normal of the interface between the two media. Figure 2 illustrates this first law of refraction. On the other hand the paraxial approximation consists in assuming that all rays lie close to the optical axis. Using the sine expansion:

$$\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \cdots \tag{2}$$

then $\phi \approx 0 \Longrightarrow \sin\phi \approx \phi$. Equation 1 becomes:

$$n_1\theta_1 \approx n_2\theta_2 \tag{3}$$

This approximation is the basis of Gauss optics or first order optics.

The quantification of the aberrations of an optical system is done by computing the difference between the real image (i.e. the one that stems from Equation 1), and the image that results from the paraxial approximation. In other words, two ray traces emerging from the same point on the object with the same angle, one exact and one approximated[1], will strike the image plane at different positions. These correspondence errors, averaged over a whole set of distinct rays, could provide a convenient basis for building a quality measure.

Finally, in the sine expansion of Equation 2, it is interesting to note that if we also consider the second term, than we obtain what is called third order optics.

[1]The approximative ray trace is virtual and computed with Gauss optics.

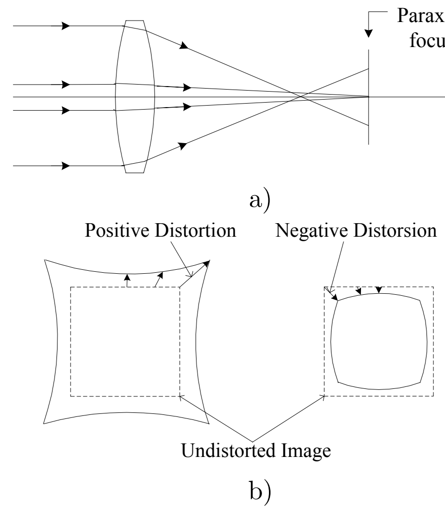

Figure 3: Illustration of: a) Spherical Aberration, and b) Distortion.

The difference between first and third order optics represents the five Seidel aberrations: spherical aberration, coma, astigmatism, field curvature, and distortion (O'Shea, 1985). Figure 3 illustrates two of these. The spherical aberration (Figure 3a) is caused by the fact that, for spherical lenses, rays coming from infinity and parallel to the optical axis do not converge to the same focus point, depending on the ray distance from the optical axis. The result of this type of aberration is a blurred image. Another type of aberration is distortion, that causes pincushion (positive distortion) or barrel (negative distortion) shaped images, as shown in Figure 3b.

3 EXISTING METHODS

Modern design of lens systems is generally done with specialized CAD softwares that help designers to visualize the lens system, evaluate its quality following precise criteria, and locally optimize the system's variables. This optimization is often done by using local search algorithms like the Damped Least Square (DLS) method. But the typical search space of optical system design is a complicated multidimensional space comprising several peaks, non-linearities and strong correlation between parameters (Sturlesi and O'Shea, 1990). Hence, a local search explores only the immediate neighborhood of the initial solution, making the result very dependent on the competence and experience of the designer. But since the beginning of the 1990's, some applications of global search methods have been made in optical design. A few researchers have successfully used simulated annealing for optical design (Forbes and Jones, 1990; Hearn, 1990). Others have modified local optimization algorithms, like the DLS algorithm, to allow exploration beyond local optima (Isshiki, 1998). These two approaches have been recently integrated in some optical CAD tools.

But not much work has been done using Evolutionary Algorithms (EA). As far as we know, only (Ono et al., 1998) have designed some lens systems using real-coded genetic algorithms. They were able to automatically design lens systems made of more than 10 parts, for some imaging applications. They also experimented with multi-objective optimization of optical systems by using the Pareto optimal selection strategy. Other research on application of EA to optical systems includes (Ben Hamida et al., 1999), which use two evolutionary approaches to design an optical surface, and (Nosato et al., 2001), that use EA for the automatic alignment of optical devices.

4 MONOCHROMATIC QUARTET PROBLEM

To evaluate the capability of our approach for lens system design, we chose a problem stated in the 1990 International Lens Design Conference (ILDC). This conference, held every four years, includes a friendly design competition for its participants. The 1990 problem (O'Shea, 1990) became a benchmark to evaluate the performance of optimization algorithms for lens system design because the 11 best solutions proposed by human experts make up only two different classes of similar solutions, and the organizers concluded that these solutions appear to be global optimums of the solution space.

The problem is named the *Monochromatic Quartet*. Essentially, it consists in finding an optical system made of four spherical lenses. Here is the formal statement of the problem (O'Shea, 1990).

> Design a 4-element, f/3, 100 mm effective focal length lens of BK7 glass, illuminated by helium d wavelength (i.e., $n = 1.51680$). The object is at infinity, the object field covers 30° full field (15° semi-field angle) and the image field is flat.
>
> Constraints on the construction includes: only spherical surfaces, no aspherics, GRIN elements, Fresnel lenses, binary elements, holographic optical elements, etc. The minimum glass thickness is 2 mm, but there is no upper limit on the size of the lens. The distortion must be less than 1% and there should be no vignetting. The last is intended to assure that vignetting could not be used to improve the edge performance on the lens. No requirement is put on the location of the stop of the system.
>
> The merit function consists of the average of the RMS blur spot for three fields : on-axis, 10.5°, and 15°, weighted equally.

The f-number (also written f/#) measures the light-collecting ability of the lens system. An effective focal length for a lens system is similar to the focal length of an equivalent single lens. The focal length itself is the inverse of the lens power, which is the capacity of making rays converge over short distances. The BK7 glass is just an ordinary type of glass frequently used for lens fabrication. The helium d wavelength constraint specifies that the problem is monochromatic, that is the considered wavelength is fixed and thus the refractive indexes are also fixed (otherwise we would have to consider the so-called chromatic aberrations). This system must not have vignetting, i.e. the image must not be truncated. It is also possible to include a stop, that is an aperture in the optical system which limits the amount of light in the system, allowing to reduce aberrations. Its diameter can be linked directly with the effective focal length and the f-number.

The error measure of the problem seeks to separate distortion from other types of aberrations. The problem statement specifies that distortion must not exceed 1% and thus implies that below this level, one should only concentrate on other aberrations. Using exact computations (Equation 1), the RMS blur spot method traces several parallel rays at a given entrance angle. These angles must be set successively at 0°, 10.5°, and 15° as specified by the problem statement. Using paraxial approximation, all the rays with the same entrance angle converge at a single point. But with exact ray traces, they will strike the image plane at different points, generally in the neighbourhood of the approximate point, and form a so-called blur spot, as illustrated in Figure 4. The RMS blur spot is computed from the variances of the position at the image plane of different exact rays with the same entrance angle. A reference ray traced with the paraxial approximation is used to evaluate the distortion, by measuring its distance from the centroid of the exact rays at the image. For more details, the interested reader is referred to (Lambda Research Corporation, 2001).

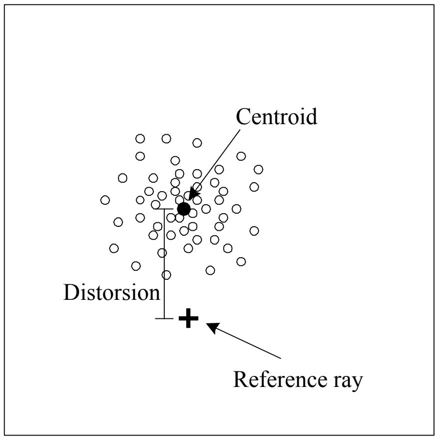

Figure 4: Illustration of the Blur Spot Measure

Table 1: Monochromatic Quartet Lens System Encoding with GA. Units for thickness, distance and stop location are mm. Units of curvatures are mm^{-1}.

Type of parameter	**# of par.**	**Encoding**	**Bounds**
Curvature	7	16 bits	$[-0.04, 0.04]$
Thickness	4	16 bits	$[2, 250]$
Distance	3	16 bits	$[0, 250]$
Stop Location	1	18 bits	$[0, 1000]$

5 LENS SYSTEM DESIGN WITH GA

As a first trial, we tried to design a lens system that meets the monochromatic quartet criteria using classical GA, with bit string representations of fixed length (Holland, 1975). The lens system modeling is straightforward and the problem has a total of 15 parameters to optimize. Table 1 summarizes the encoding and the upper/lower bounds for each parameter encoded in the bit strings. The chosen bounds are large enough to allow the exploration of all physically feasible solutions.

The value of the last curvature and aperture stop are not included in this table because these parameters are not evolved. They are set so that the lens system respects the problem specifications (effective focal length and f-number). The distance between the last surface and the image plane is also calculated in order that approximative rays having the same field angle focus on the image plane. Also, the lens systems are validated during the evolution to ensure that they are physically possible (to prevent lens overlap, etc.). When impossible configurations appear, the problematic lens diameters, distances between lenses or lens thicknesses are corrected until the system become physically feasible.

Table 2: Evolution Parameters with GA

Evolution parameter	**Value**
Population size	5000
Number of generations	1000
Crossover probability	0.03
Mutation probability	0.01
Participants to tournament selection	3

To implement this GA, we used a C++ framework for evolutionary computations named Open BEAGLE (Gagné and Parizeau, 2002). Also a C++ library, named *Library for Lens System Ray Tracing* (Gagné and Beaulieu, 2001), was developed to compute ray tracing through a given lens system. This library facilitates the fitness measure calculation, that needs to evaluate several exact and approximative ray traces.

For the fitness measure, we defined the following equation:

$$Fitness = \begin{cases} \dfrac{1.0}{1.0 + RMS} & \%_{dist} \leq 1.0 \\ \dfrac{1.0}{\%_{dist}} \times \dfrac{1.0}{1.0 + RMS} & \%_{dist} > 1.0 \end{cases}$$

where $\%_{dist}$ is the maximum percentage of distortion observed, and RMS is the average value of the RMS blur spot for the three initial field angles mentioned in the problem statement. This fitness equation provides a measure spectrum, normalized between 0 and 1, that is detailed enough to adequately differentiate individuals. It also ensures that the lens system having more than 1% of distortion are sufficiently penalized during the evolutions. Experimentally, we observed that all the best solutions obtained do not have more than 1% of distorsion.

Preliminary tests were first conducted in order to obtain a good idea of evolution parameters to use for this problem. The used parameters are presented in Table 2. Then, 30 evolution runs were conducted using these parameters, each needing about 2 to 3 hours of CPU time on an Athlon processors running at 1.2 GHz (the runs were conducted in parallel on a Beowulf cluster of 25 nodes).

6 RESULTS USING GA

The best solution obtained with GA produces an averaged RMS blur spot of 0.0019 mm, compared with 0.0024 mm for the best human result reported at the 1990 ILDC monochromatic quartet contest. This RMS

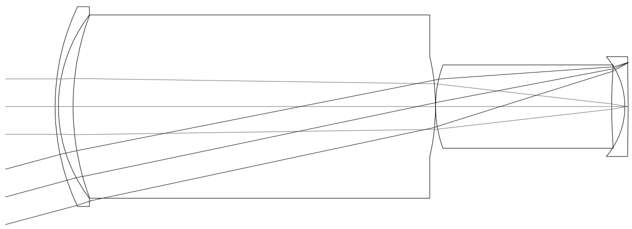

Figure 5: Best Monochromatic Quartet Presented at the 1990 ILDC

Table 3: Parameters of the Best Monochromatic Quartet Presented at the 1990 ILDC

Radius	**Thickness**	**Aperture**	**Glass**
140.0	2.0	60.0	BK7
90.6	8.8	55.0	air
155.86	206.8	55.0	BK7
0.0	**11.2**	**13.757654**	**BK7**
-134.7	0.05	30.0	air
72.6	106.0	25.0	BK7
357.38	7.9	25.0	air
-45.82	2.0	30.0	BK7
-1458.1	0.1	30.0	air

(Bold surface is the aperture stop.)

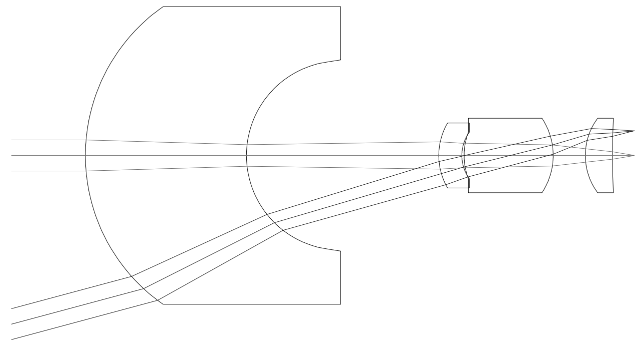

Figure 6: Best Design Found with GA

Table 4: Parameters of the Best Design Found with GA

Radius	**Thickness**	**Aperture**	**Glass**
194.6968	173.638	160.0	BK7
101.6046	207.408	160.0	air
69.2262	24.8946	35.0	BK7
42.8546	2.43763	25.0	air
70.8426	49.7265	25.0	BK7
0.0	**46.3269**	**12.172236**	**BK7**
-72.1277	34.6571	40.0	air
66.8914	29.2579	40.0	BK7
834.9267	23.7954	40.0	air

(Bold surface is the aperture stop.)

blur spot is the average of three statistical measures of many exact rays scattering from the three given entrance angle. The RMS value is dependent on both these entrance angles, but also on the choice of the computed rays for each angle. Moreover, when the RMS is very small, as in this case, it is not highly accurate due to rounding errors and image plane placement. Varying the image plane position may lower a little bit the RMS. The RMS measures reported here have been calculated with the well known CAD tool OSLO (Lambda Research Corporation, 2001), in order to eliminate any bias that our own RMS measure might introduce. Using our own RMS measure could have favored the evolved systems relatively to the human designed ones.

The best system evolved with GA is thus 23% better than the best presented at the 1990 ILDC (following the OSLO RMS measure). Figure 5 and Table 3 present the best 1990 ILDC design, while Figure 6 and Table 4 present the best GA design.

7 LENS SYSTEM DESIGN AND RE-ENGINEERING WITH GP

As a second trial, the same monochromatic quartet problem was tackled, using the same EC environment (Open BEAGLE), but this time specialized for GP. The evolving genetic programs represent some modifications to apply on a given initial lens system. The evolving programs are made of three types of primitives. The first type includes primitives that can increment/decrement an iterator that points to a lens surface. The second type is composed of primitives that modify a parameter of the current lens surface. The other type includes classic arithmetic operations. The terminals of the genetic trees are ephemeral constants (Koza, 1992), randomly generated between -1 and 1. Two ADFs (Koza, 1994) were also added to favor emergence of building blocks, which can be useful for this type of problem. Table 5 presents the complete set of primitives. Note that we intentionally encouraged the search to be in the initial solution neighborhood by allowing mostly small parameter modifications.

To evaluate each individual, an iterator is affected to the first surface of the initial lens system. Once the system is modified, it is validated to ensure that it respects the problem specifications and that it is physically feasible, as explained in section 5. Thereafter, the

Table 5: GP Primitives Used to Evolve Lens Systems

Primitives	**Inputs**	**Description**
`First`, `Last`	1	Set the iterator to the first/last surface.
`Next`, `Prev`	1	Iterate to the next/previous surface.
`Curv+`, `Dist+`	1	Add $0.1 \times \Delta \times input$ to the surface curvature/distance value[a].
`Curv*`, `Dist*`	1	Multiply the surface curvature/distance value with $1.0 + input$.
`Stop+`, `Stop*`	1	Add to/multiply the stop location value (as with the curvature/distance).
`+`, `-`, `*`, `/`	2	Add, subtract, multiply, or divide two floating-point numbers.
`Ephemerals`	0	Randomly generated constants in $[-1, 1]$.

[a]The Δ term represents the maximum value for the corresponding variable. For curvature $\Delta = 0.04$, for distance $\Delta = 250$, and for stop location $\Delta = 1000$ (see Table 1).

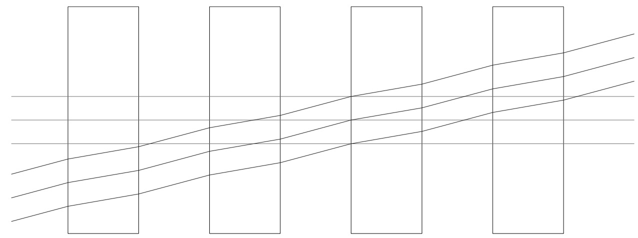

Figure 7: Raw System Used for Lens System Design with GP

Table 6: Parameters of the Raw System Used for Lens System Design with GP

Radius	**Thickness**	**Aperture**	**Glass**
0.0	50.0	80.0	BK7
0.0	50.0	80.0	air
0.0	50.0	80.0	BK7
0.0	50.0	80.0	air
0.0	**50.0**	**80.0**	**BK7**
0.0	50.0	80.0	air
0.0	50.0	80.0	BK7
0.0	50.0	80.0	air

(Bold surface is the aperture stop.)

lens system fitness is calculated. The fitness measure is the same as the one used for GA evolutions.

Two different approaches were applied to solve the monochromatic quartet problem with GP. Firstly, we tried the re-engineering of good solutions by using solutions obtained with GA as initial lens systems for the evolving process. Secondly, we tried the re-engineering using a raw lens system made of four "lenses" of zero curvature (see Figure 7 and Table 6). With this approach, the capacity of the GP to design a lens system from scratch is evaluated.

Using GP this way is interesting because the representation isolates the genotype from the phenotype (Gruau, 1994; Koza et al., 1999). The evolved programs (the genotypes) modify the initial lens system to spawn better ones (the phenotypes).

Again, preliminary tests were made to find adequate evolution parameters (see Table 7). For the raw system as a starting point, 16 differents runs were conducted, while 8 runs were made for the re-engineering of each of the 5 lens system designs selected from the GA results. The chosen GA designs are a representative set of results with GA, i.e. we chose some bad, good and very good lens systems in order to compare the re-engineering in each case. Each evolution needed an average of 36 hours for design and 3 hours for re-engineering on a single Athlon 1.2 GHz processor.

8 RESULTS USING GP

Table 8 presents the best GP re-engineering results for the 5 chosen GA solutions. Note that the best GA design (GA-1) is still the best result after the GP re-engineering, with a RMS blur spot of 0.0016 mm. This overall best result is 34% better than the best 1990 ILDC design. Table 9 presents the parameters of this overall best lens system. For all the best GP re-engineering, the topology remains unchanged from the initial lens system. This indicates that the GP re-engineering is probably doing a local search.

For the GP re-engineering using the raw system as starting point (design from scratch), the best result was an RMS blur spot of 0.0039 mm, which is 60% worse than the best 1990 ILDC design. The corresponding design is presented in Figure 8 and Table 10.

Table 7: Evolution Parameters with GP

Evolution parameter	**Value**
Population size	5×1000
Migration type	Unidirectional ring
Number of migrants	10
Number of generations	1000 (re-engineering) 5000 (design)
Crossover probability	0.9
Swap mutation probability	0.5
Participants to tournament selection	3
Initial tree height	$[4, 7]$
Maximum tree height	20

Table 8: Best Results Obtained with GP (Re-Engineering)

Case	**Original RMS (mm)**	**Re-engineered RMS (mm)**	**RMS Shift**
GA-1	0.0019	0.0016	-14%
GA-2	0.0084	0.0071	-16%
GA-3	0.0187	0.0139	-25%
GA-4	0.0308	0.0105	-66%
GA-5	0.0909	0.0843	-7.3%

Table 9: Parameters of the Best Re-Engineering Design Found with GP

Radius	**Thickness**	**Aperture**	**Glass**
194.6968	173.638	160.0	BK7
101.5659	207.496	160.0	air
68.9403	24.829	35.0	BK7
42.8546	2.42726	25.0	air
70.8426	44.7725	25.0	BK7
0.0	**51.2809**	**12.210297**	**BK7**
-72.0648	34.6571	40.0	air
66.4712	31.4814	40.0	BK7
948.4696	21.9564	40.0	air

(Bold surface is the aperture stop.)

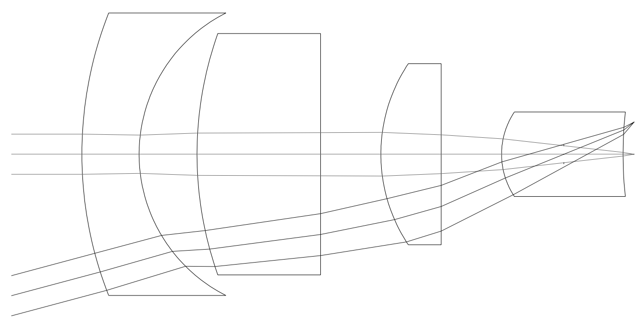

Figure 8: Best Design Found with GP (Using the Raw System as Starting Point)

9 DISCUSSION

Results show that GA and GP are capable of high quality lens system design. Indeed, they have spawn comparable or even better solutions (in terms of RMS blur spot) than the best designs presented by human experts. Thus they meet one of the criteria for human-competitiveness in evolutionary computations (Koza et al., 2000).

The best obtained result, however, has a topology that differs significantly from the two best system classes presented at the 1990 ILDC. Thus, the evolving process has probably discovered a new optimum topology class for the monochromatic quartet problem. The best obtained RMS blur spots with GA design and GP re-engineering are respectively 23% and 34% better than the best previously reported human result (1990 ILDC).

The search space in lens system design is very complex and comprises correlations between the different parameters. This makes the lens system design difficult with GA because good schemes of parameters that are far together on the bit string are likely to be destroyed by the crossover operation. For the lens system design with GP, we intentionally restricted the search in the initial system neighborhood. This has probably disfavored the convergence toward optimums that were situated far from the starting point.

10 FUTURE WORKS

For future works, we plan to experiment with an hybrid GA-GP approach, to benefit from the capacity of GA to optimize numerical parameters, using a GP variable length representation. We also plan to use a multi-objective merit function using different types of Seidel aberrations (see (O'Shea, 1985) for details) and lens costs as evolving criteria. Furthermore, we could use a database of existing commercial lenses, with associated prices to simulate real life design situations.

We will also study more deeply the re-engineering of good solutions for lens system design and for other applicative contexts. We will try to develop methods that make good compromises between local and global optimization, to enable sufficient search space exploration that facilitates the discovery of the best solu-

Table 10: Parameters of the Best Design Found with GP (Using the Raw System as Starting Point)

Radius	**Thickness**	**Aperture**	**Glass**
318.8959	47.4182	117.0	BK7
131.18	47.9761	117.0	air
298.4896	102.302	100.0	BK7
0.0	50.0	100.0	air
135.1501	50.0	75.0	BK7
0.0	50.0	75.0	air
62.973	51.5426	35.0	BK7
0.0	**49.1266**	**7.029558**	**BK7**
334.1364	9.3196	35.0	air

(Bold surface is the aperture stop.)

tions. Finally, we expect to integrate cultural evolving aspects (Spector and Luke, 1996) to define new generic evolutionary way of proceeding for re-engineering.

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References

Ben Hamida, S., Racine, A., and Schoenauer, M.: 1999, Two evolutionary approaches to design phase plate for tailoring focal-plane irradiance profile, in *Proceedings of Artificial Evolution 1999*, Vol. 2210 of *LNCS*, pp 266–276, Springer Verlag

Forbes, G. W. and Jones, A. E. W.: 1990, Towards global optimization with adaptive simulated annealing, in *Proceedings of International Lens Design Conference 1990*, Vol. 1354 of *SPIE*

Gagné, C. and Beaulieu, J.: 2001, *Library for Lens System Ray Tracing*, http://www.gel.ulaval.ca/~cgagne/llsrt

Gagné, C. and Parizeau, M.: 2002, Open BEAGLE: A new C++ evolutionary computation framework, in *Genetic and Evolutionary Computations COnference (2002)*, New York, NY, USA

Gruau, F.: 1994, *Neural Network Synthesis using Cellular Encoding and the Genetic Algorithm, Ph.D. thesis*, Laboratoire de l'Informatique du Parallélisme, Ecole Normale Supérieure de Lyon, France

Hearn, G. K.: 1990, Pratical use of generalized simulated annealing optimization on microcomputers, in *Proceedings of International Lens Design Conference 1990*, Vol. 1354 of *SPIE*

Holland, J. M.: 1975, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor, MI

Isshiki, M.: 1998, Global optimization with escape function, in *Proceedings of International Optical Design Conference 1998*, Vol. 3482 of *SPIE*

Koza, J. R.: 1992, *Genetic Programming: On the Programming of Computers by Means of Natural Selection*, MIT Press, Cambridge, MA, USA

Koza, J. R.: 1994, *Genetic Programming II: Automatic Discovery of Reusable Programs*, MIT Press, Cambridge, MA, USA

Koza, J. R., David Andre, Bennett III, F. H., and Keane, M.: 1999, *Genetic Programming 3: Darwinian Invention and Problem Solving*, Morgan Kaufman

Koza, J. R., Keane, M. A., Yu, J., Bennett III, F. H., and Mydlowec, W.: 2000, *Automatic Creation of Human-Competitive Programs and Controllers by Means of Genetic Programming*, *Genetic Programming and Evolvable Machines* pp 121–164

Lambda Research Corporation: 2001, *OSLO: Optics Software for Layout and Optimization (Reference Manual)*, Lambda Research Corporation

Nosato, H., Kasai, Y., Itatani, T., Murakawa, M., Furuya, T., and Higuchi, T.: 2001, Evolvable optical systems and their applications, in *Evolvable Systems: From Biology to Hardware (Proc. of ICES 2001)*, Vol. 2210 of *LNCS*, pp 327–339, Springer Verlag

Ono, I., Koboyashi, S., and Yoshida, K.: 1998, Global and multi-objective optimization for lens design by real-coded genetic algorithms, in *Proceedings of International Optical Design Conference 1998*, Vol. 3482 of *SPIE*

O'Shea, D. C.: 1985, *Elements of Modern Optical Design*, John Wiley and Sons

O'Shea, D. C.: 1990, The monochromatic quartet: A search for the global optimum, in *Proceedings of International Lens Design Conference 1990*, Vol. 1354 of *SPIE*

Spector, L. and Luke, S.: 1996, Cultural transmission of information in genetic programming, in *Genetic Programming 1996: Proceedings of the First Annual Conference*, pp 209–214, MIT Press

Sturlesi, D. and O'Shea, D. C.: 1990, Future of global optimization in optical design, in *Proceedings of International Lens Design Conference 1990*, Vol. 1354 of *SPIE*