Evolutionary Design of Digital Filters With Application to Subband Coding and Data Transmission

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***Abstract*—In this paper, two evolutionary programming (EP) algorithms (classical EP and fast EP) are applied to design prototype lowpass Finite Impulse Response filters for use in a modulated filterbank. The chosen filter design technique is based on frequency-sampling (where the Fourier transform magnitudes of the filter are the objective variables). Design is simplified by constraining most of these values, leaving only a small number of values in the filter transition band to be optimized. The EP algorithms were used to determine the optimum values for this subset of values. Since there is an additional monotonic constraint on the transition band values, a modification to the EP algorithms was developed called variable limits evolutionary programming. Results indicate that a) both EP algorithms were insensitive to initial conditions, and reliably found the minimum values of the chosen objective functions, and b) the designed prototype filters are suitable to obtain near-perfect reconstruction filter banks, offering quality parameters comparable or better than systems obtained using other techniques.**

***Index Terms*—Channel bank filters, evolutionary programming, filter bank design, filtering theory, modified-DFT filter bank, modulated filter bank, multicarrier modulator, nearly perfect reconstruction.**

I. Introduction

MULTIRATE Systems are used in a wide range of applications, from data compression (speech, audio, image or biosignals coding) to data transmission [1]. An important subclass of filter banks (FBs) is the modulated group, in which all the analysis and synthesis filters are obtained by the modulation of low-pass prototype filters [2]. This kind of filter bank was originally applied in subband coding, but recently such filter banks have been proposed in the design of multicarrier communications transceivers as an alternative solution to the transmitting and receiving stages in orthogonal frequency division multiplexing (OFDM) or discrete multitone modulation (DMT). These systems can be simply reexpressed as discrete fourier transform (DFT) filter banks, and due to the underlying properties of rectangular windowing, they have relatively poor selectivity and discrimination between subchannels. This can lead to significant performance deterioration through intercarrier interference. For this reason, other kinds of multicarrier modulators (MCM) have been proposed [3]–[5]. Since many of these approaches rely on the design of an appropriate prototype filter, the design of such prototype filters for modulated filter banks (MFBs) is a topic that continues to receive widespread attention (see, for example, references included in [6]–[9]).

In this paper, we apply evolutionary programming (EP) to the design of arbitrary-length filters for multirate applications. EP is a population based heuristic, which was first proposed in the field of artificial intelligence [10]. EP is basically a search algorithm which uses random mutation for exploring the search space. In the classical EP algorithm, this mutation is based on a Gaussian distribution. However, in recent years EP has been more extensively studied [11]–[13], and several variations of the EP algorithm have been proposed, based on different probability distribution mutations such as Cauchy or Levy distributions [14]–[16].

EP has been applied to many different problems in engineering and sciences, including telecommunications [17], industrial applications [18]–[20], computer science [21], circuits and machinery design [22], [23], environmental [24], [25], and biomedical applications [26], [27]. It has also been applied to the design of filters. For example, in [28] Görne and Schneider applied an evolutionary algorithm to the design of digital filters. In another work, White and Flockton [29] compared the performance of different nature-inspired algorithms such as evolutionary programming and simulated annealing, to the design of infinite impulse response (IIR) filters. They demonstrated that the evolutionary approach to the problem obtained good designs which were comparable with results from the simulated annealing algorithm. Other works which considered the design of filters using the evolutionary-based techniques are [30]–[36].

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In this paper, we propose the application of EP to the design of filters for multirate applications, such as the design of prototype filters for FBs or Multicarrier Modulators (MCM). To our knowledge, this application has not been previously explored with this technique. The proposed method is based on the frequency sampling approach for designing FIR filters [37], [38], and it is highly efficient due to the very low number of parameters which has to be optimized. A similar technique for obtaining linear-phase filters of restricted length has been proposed in [8]. Recently, the general expressions that allow us to obtain arbitrary-length filters using a similar approach has been presented in [9]. Previously, the algorithms used for optimizing the parameters of these filters (or to be precise, the magnitude response values of all the samples in the transition band) can be considered in two categories.

a) Multidimensional unconstrained nonlinear minimization algorithms (Nelder-Mead). Previous experimental results have shown however that success is strongly dependent on the initial values chosen for running the algorithms, and that is the reason why two different functions have been proposed in [8] and [9] for selecting the initial values in the transition band.
b) Using a standard implementation of a quasi-Newton algorithm, which uses the gradient in the line search procedure. Since, in general, no function gradients are available, they must be numerically estimated via finite differences at the cost of additional function evaluations during the inner iterations of the line search procedure. This implementation of the method with a gradient-based line search has the advantage of being theoretically sound and guaranteed to terminate, though sometimes at the expense of requiring a large number of evaluations to estimate the gradient.

The main contributions of this paper are the following. We propose a modification of the EP algorithm for solving the design of arbitrary-length filters for multirate applications. Basically, the EP algorithm must be modified to deal with the restrictions in the variables to be optimized (the samples of the magnitude response in the transition band), requiring the introduction of a novel evolutionary programming algorithm called variable limits evolutionary programming (see Section V). This approach is a way of tackling problems which need monotonicity in their solutions, and it is much more robust than the standard optimization tools such as Nelder-Mead or quasi-Newton, even if multiple initializations are considered. Also, we will show through experimental results, that our approach leads consistently to the best minimum solution of the problem cost function. In addition, the proposed algorithm is not sensitive to the choice of the initial conditions at algorithm start-up, which is a significant improvement with respect to previously reported techniques.

Following Sections I and II, the rest of the paper is structured as follows. Section III defines the problem, and Section IV presents the general optimization procedure, and reviews previous related works in order to show how to define the cost function to be minimized. In Section V, we present our evolutionary programming approach to the problem, introducing a modification of the classical evolutionary programming and the fast evolutionary programming algorithms [14], for adapting them to the problem of filters design. In Section VI, we show through experimental results, the need to solve the optimization problem by means of a powerful algorithm. We also show that the performance of the designed systems is extremely good as compared to other design techniques. Finally, Section VII summarizes our conclusions.

II. Range of Application

In this section, we briefly show the utility of our proposed method through a wide range of applications where the filter designed by the proposed technique can be applied. First, although initially the conditions were focused on obtaining linear-phase filters that are approximately a spectral factor of a $2M$-band filter [8], [9], we can change the initial values of the samples to be optimized and define an appropriate cost function to be optimized in order to obtain multirate interpolation and decimation filters. For this reason, this kind of filter appropriately designed could be applied, for example, to decimation in radio receivers or for $\Sigma\Delta$ A/D converters. The main advantage of the filter designed with the proposed technique is as follows. Since by definition, there are many samples whose value is null, the designed multirate filter has the same property, offering the same number of transmission zeros as null samples have been previously defined in the initial conditions.

With regard to modulated filter banks, it is well known that prototype filters valid for M-channel NPR CMFB can be also used to design several kinds of $2M$-channel complex modulated filter banks (CxMFBs) (for a detailed description of prototype filters equivalence for different families of filter banks, see [39] and [40]). In this way, the possible applications of the proposed method can be extended for a wide variety of systems, as is shown in Table I, with a brief summary of conventional and recent applications.

III. Problem Definition

Let $P(e^{j\omega})$ be the frequency response of the finite impulse response (FIR) filter to be designed. Let $P[k] \equiv P(e^{j\omega_k})$ be its samples at N points uniformly spaced in the interval $[0, 2\pi)$, where $\omega_k = (k+\alpha) \times 2\pi/N$, $0 \le k \le (N-1)$, and $\alpha = 0$ or $\alpha = 1/2$. The relation between the impulse response coefficients and the frequency response samples is given by

$$p[n] = \frac{1}{N} \sum_{k=0}^{N-1} P(e^{j\omega_k}) e^{j\frac{2\pi}{N}(k+\alpha)n} \tag{1}$$

where $0 \le n \le (N-1)$ [38].

For simplicity, we only focus on the design of low-pass filters. The initial values of $P[k]$ must be appropriately defined before optimization. The number of samples L in the transition band should be selected $(L \ge 1)$, and the sample index r nearest to the transition band center must be obtained. When L is an even number and there are two index samples equidistant from the

TABLE I
APPLICATIONS OF MODULATED FILTER BANKS

Filter Bank	**Number of Channels**	**Applications**	**References**
Cosine Modulated	M	Data transmission; subband coding	[3], [4]
Sine Modulated	M	Narrowband interference detection and suppression in spread spectrum systems	[41]
Discrete Fourier Transform (DFT)	$2M$	Power systems harmonic measurement; data transmission via OFDM and DMT (also for multipath fading channels or radio frequency interference suppression)	[42], [43], [44]
Generalized DFT	$2M$	Subband adaptive filtering	[45]
Modified DFT	$2M$	Subband coding; data transmission	[40], [46]
Exponentially Modulated	$2M$	Subband coding; data transmission	[5], [47]

transition band center, the smaller number should be chosen for r. Therefore, given L, r, and N, the values of the magnitude response $|P[k]|$ and the phase response $\arg\{P[k]\}$ are defined as

$$|P[k]| = \begin{cases} 1, & 0 \le k \le r - \lceil L/2 \rceil \\ & \text{(passband)} \\ f(k) & r - \lceil L/2 \rceil + 1 \le k \le r + \lfloor L/2 \rfloor, \\ & \text{(transition band)} \\ 0 & r + \lfloor L/2 \rfloor + 1 \le k \le \lfloor (N-1)/2 \rfloor \\ & \text{(stopband)} \end{cases} \tag{2a}$$

$$|P[N-1-k]| = |P[k]|, \lfloor (N-1)/2 \rfloor + 1 \le k \le (N-1) \tag{2b}$$

$$\arg\{P[k]\} = \begin{cases} -\frac{(N-1)}{2} \cdot \frac{2\pi}{N} \cdot (k+\alpha) \\ \qquad 0 \le k \le \lfloor (N-1)/2 \rfloor \\ \frac{(N-1)}{2} \cdot \frac{2\pi}{N} \cdot (N-(k+\alpha)) \\ \qquad \lfloor (N-1)/2 \rfloor + 1 \le k \le (N-1) \end{cases} \tag{2c}$$

where $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ denote rounding to the next smaller or larger integer, respectively.

Moreover, when the filter is intended as a prototype for MFBs, a good choice is to locate the transition band center at approximately the frequency $\pi/(2M)$, and the closest value to $\omega_c = \pi/(2M)$ is defined as $\omega_r = (r+\alpha)2\pi/N$, $r \in Z^+$. The parameter M is related to the number of channels in the filter bank. In the general case, there is no predefined relationship between M and N, though in practice cases, $N = kM + 1$ or $N = kM$ values are frequently used.

The function $f(k)$ defined in (2) allows us to write the magnitude values of the transition band samples. In previous works [8], [9], the optimization process was carried out successfully after a careful selection of these initial values. However, in Section V, we propose an evolutionary programming-based algorithm that ensures fast convergence to optimum solutions without the need for special care in the choice of the initial values for the function $f(k)$.

Consequently, the problem definition consists of finding the optimum values of the frequency response samples $P_{\text{opt}}[k]$ in order to minimize an objective function ψ, whose exact definition depends on the application for which the filter is being designed.

IV. Optimization and Previous Approaches

In this section, we describe the general optimization algorithm for obtaining the filter based on the frequency sampling approach. Furthermore, in order to develop a useful definition of the objective function, we review previous designs in which the optimization problem was focussed on the design of NPR CMFBs.

A. Optimization Algorithm

Once the initial values have been appropriately chosen, the optimization procedure for a low-pass filter consists of the following steps:

1) Select the filter length N.
2) Select the required number L of samples in the transition band.
3) Initialize the N samples of the frequency response as described before in (2). The resulting vector $|\mathbf{P}[k]|$, with corresponding magnitude response initial values, is made up as follows:

$$|\mathbf{P}[k]| = \Big[\underbrace{1 \;\cdots\; 1}_{\text{passband}} \quad \underbrace{f[q+1] \;\cdots\; f[q+L]}_{\text{transition band}} \quad \underbrace{0 \;\cdots\; 0}_{\text{stopband}}\Big]$$
$$0 \le k \le \lfloor (N-1)/2 \rfloor \quad q = r - \lceil L/2 \rceil. \tag{3}$$

4) Let $\mathbf{f} = [f[q+1]\; f[q+2] \cdots f[q+L]]$ be the vector whose elements are the samples of the magnitude response at the transition band. Find $\mathbf{f}_{\text{opt}}$, i.e., determine the values of the components of $\mathbf{f}$ that minimize an objective function ψ.
5) Calculate the optimum values of the frequency response samples $\mathbf{P}_{\text{opt}}[k]$. These values are obtained from (2) and

(3), by replacing the initial values of the magnitude response in the transition band in (2) by the optimized values obtained in the previous step:

$$|\mathbf{P}_{\text{opt}}[k]| = \Bigg[\underbrace{1\,1\,\cdots\,1}_{\text{passband}}\quad \underbrace{f_{\text{opt}}[q+1]\,\cdots\,f_{\text{opt}}[q+L]}_{\text{transitionband}}\quad \underbrace{0\,\cdots\,0}_{\text{stopband}}\Bigg] \quad 0 \le k \le \lfloor (N-1)/2 \rfloor \tag{4a}$$

$$|\mathbf{P}_{\text{opt}}[N-1-k]| = |\mathbf{P}_{\text{opt}}[k]| \quad \lfloor (N-1)/2 \rfloor + 1 \le k \le (N-1). \tag{4b}$$

6) Using $P_{\text{opt}}[k]$ obtain the filter coefficients $p_{\text{opt}}[n]$ through

$$p_{\text{opt}}[n] = \frac{1}{N}\sum_{k=0}^{N-1} P_{\text{opt}}(e^{j\omega_k})e^{j\frac{2\pi}{N}(k+\alpha)n}. \tag{5}$$

B. Objective Functions ψ for Modulated Filter Banks

Given a choice of system design, the objective function ψ to be optimized in step 4 above has to be defined. For example, for MFBs it is well known the following conditions:

$$\left|P(e^{j\omega})\right| \approx 0 \quad |\omega| > \pi/M \tag{6}$$

$$\left|T(e^{j\omega})\right| = \left|\sum_{i=0}^{M-1} F_i(e^{j\omega}) \cdot H_i(e^{j\omega})\right| \approx 1 \tag{7}$$

allow us to obtain approximate reconstruction in the M-channel cosine-modulated filter bank and the $2M$-channel complex modulated filter banks. In (6) and (7), $H_i(e^{j\omega})$ and $F_i(e^{j\omega})$ are, respectively, analysis and synthesis (or receiving and transmitting for data transmission applications) filters obtained by modulation from the designed prototype filter $P(e^{j\omega})$.

In previous works, the objective function ψ to be minimized for designing MFBs has been defined in different ways, and there are a multitude of useful definitions that could be proposed. For example, three useful definitions are as follows.

a) As defined in [6]

$$\psi = \max_{\omega}\left\{\left|P(e^{j\omega})\right|^2 + \left|P\left(e^{j(\omega-\pi/M)}\right)\right|^2 - 1\right\} \tag{8}$$

where $\omega \in (0, \pi/M)$. This function guarantees that the resulting systems closely satisfies the perfect reconstruction (PR) property.

b) Lin and Vaidyanathan define in [7] a different cost function

$$\psi = \max_{n, n\neq 0} |g[2Mn]| \tag{9}$$

where $G(e^{j\omega})$ is the discrete-time Fourier Transform of $g[n]$, defined as $G(e^{j\omega}) = |P(e^{j\omega})|^2$.

c) A third example of cost function that can be minimized for modulated filter banks is

$$\psi = \frac{\int_{\frac{\pi}{2M}}^{\pi} P(e^{j\omega})d\omega}{\int_0^{\frac{\pi}{2M}} P(e^{j\omega})d\omega}. \tag{10}$$

Therefore, efficient prototype filters for MFBs can be obtained using these cost functions, or suitable alternatives.

V. Evolutionary Programming for Multirate Filters Design

As we explained in Section I, the main problem in previous work on frequency-sampling based design was the selection of appropriate initial values for the $f(k)$ samples to be optimized. In [8], the function

$$f(k) = \left(\frac{\omega_s - k\cdot 2\cdot\pi/N}{\omega_s - \omega_p}\right) \quad q+1 \le k \le q+1+L \tag{11}$$

was proposed to generate appropriate values, where ω_p is the frequency corresponding to the last sample of the passband and ω_s is the frequency corresponding to the first sample of the stopband. For the design examples included in [9], a different function was chosen

$$f(k) = 0.95 - \left(\frac{\omega_s - (L+1-k)\cdot 2\cdot\pi/N}{\omega_s - \omega_p}\right)^2 \quad q+1 \le k \le q+1+L. \tag{12}$$

In our designs, we will use two different techniques for minimize the objective function ψ:

a) a multidimensional unconstrained nonlinear minimization (Nelder-Mead);

b) a quasi-Newton algorithm.

In Section VI we will show, by means of several examples, the importance of proper selection of the initial values in the transition band, since neither of the previous proposed $f(k)$ leads consistently to the best minimum solution of ψ, no matter the number of channels M or filter length N. This problem is solved by using two evolutionary programming algorithms as explained later. Specifically, we have implemented both a classical evolutionary programming and a fast evolutionary programming. The classical evolutionary programming algorithm (CEP) is described in the work by Bäck and Schwefel [11], [14]. It is used to optimize a given function $f(\mathbf{x})$, i.e., obtaining $\mathbf{x}_o$ such that $f(\mathbf{x}_o) < f(\mathbf{x})$, with $\mathbf{x} \in [lim\_inf, lim\_sup]$. The algorithm performs as follows.

1) Generate an initial population of μ individuals (solutions). Set $k = 1$. Each individual is taken as a pair of real-valued vectors $(\mathbf{x}_i, \sigma_i)$, $\forall i \in \{1, \cdots, \mu\}$, where $\mathbf{x}_i$s are objective variables (e.g., transition band magnitudes), and σ_i's are standard deviations for Gaussian mutations.

2) Evaluate the fitness value for each individual $(\mathbf{x}_i, \sigma_i)$ (using the defined objective function for example).

3) Each parent $(\mathbf{x}_i, \sigma_i)$, $\{i = 1, \cdots, \mu\}$ then creates a single offspring $(\mathbf{x}'_i, \sigma'_i)$ as follows:

$$\mathbf{x}'_i = \mathbf{x}_i + \sigma_i \cdot \mathbf{N_1(0,1)} \tag{13}$$

$$\sigma'_i = \sigma_i \cdot \exp\left(\tau' \cdot N(0,1) + \tau \cdot \mathbf{N(0,1)}\right) \tag{14}$$

where $N(0,1)$ denotes a normally distributed one-dimensional random number with mean zero and standard deviation one, and $\mathbf{N(0,1)}$ is a vector containing random numbers of mean zero and standard deviation one, generated anew for each value of i. The parameters τ and τ' are commonly set to $(\sqrt{2\sqrt{n}})^{-1}$ and $(\sqrt{2n})^{-1}$, respectively [14].

4) If $x_i(j) > lim\_sup$ then $x_i(j) = lim\_sup$ and if $x_i(j) < lim\_inf$ then $x_i(j) = lim\_inf$.

5) Calculate the fitness values associated with each offspring $(\mathbf{x}'_i, \sigma'_i, \forall i \in \{1, \cdots, \mu\})$.
6) Conduct pairwise comparison over the union of parents and offspring: For each individual, p opponents are chosen uniformly at random from all the parents and offspring. For each comparison, if the individual's fitness is better than the opponent's, it receives a "win."
7) Select the μ individuals out of the union of parents and offspring that have the most wins to be parents of the next generation.
8) Stop if the halting criterion is satisfied, and if not, set $k = k + 1$ and go to Step 3.

The fast evolutionary programming (FEP) is described and compared with the CEP in [14]. The FEP is similar to the CEP algorithm, but it performs a mutation following a Cauchy probability density function, instead of a Gaussian-based mutation. The one-dimensional (1-D) Cauchy density function centered at the origin is defined by

$$f_t(x) = \frac{1}{\pi} \frac{t}{t^2 + x^2} \tag{15}$$

where $t > 0$ is a scale parameter. See [14] for further information about this topic. Using this probability density function, the FEP algorithm substitutes step 5 of the CEP by the following:

$$x'_i(j) = x_i(j) + \mu_i(j)\delta_j \tag{16}$$

where δ_j is a Cauchy random variable with the scale parameter set to $t = 1$.

In this paper, we use a variation of the CEP and FEP procedures described above to obtain the optimal value of the vector $|\mathbf{P}[k]|$, whose elements correspond to the magnitude response values, previously defined in (3). As aforementioned, the components of vector $\mathbf{f}$ are the samples of the magnitude response in the transition band. This vector is such that the constraint

$$f[q+1] > f[q+2] > \cdots > f[q+L] \tag{17}$$

should also be satisfied for good performance. This requires some modification of the evolutionary programming approach. We propose a modification that we have called variable limits evolutionary programming (VLEP), which can be described in the following way (VLCEP and VLFEP).

1) Generate the initial population of μ individuals. Set $k = 1$. Each individual is taken as a pair of real-valued vectors $(\mathbf{x}_i, \sigma_i)$, $\forall i \in \{1, \cdots, \mu\}$, where $\mathbf{x}_i$s are objective variables, and σ_is are standard deviations for Gaussian mutations, as before.
2) Sort the values of $\mathbf{x}_i$ in such a way that $x_i(j) > x_i(k)$ if $j > k$.
3) Set a vector of limits for the variables, in the following way: $lim\_inf_{x(1)} = lim\_inf$, $lim\_sup_{x(1)} = x(2)$, $lim\_inf_{x(2)} = x(1)$, $lim\_sup_{x(3)} = x(4), \cdots, lim\_inf_{x(L)} = x(L-1)$, $lim\_sup_{x(L)} = lim\_sup$.
4) Evaluate the fitness value for each individual $(\mathbf{x}_i, \sigma_i)$.
5) Each parent $(\mathbf{x}_i, \sigma_i)$, $\{i = 1, \cdots, \mu\}$ creates then a single offspring $(\mathbf{x}'_i, \sigma'_i)$ using (13) for the CEP or (16) for the FEP algorithm.
6) Reset the vector of limits: $lim\_inf_{x(1)} = lim\_inf$, $lim\_sup_{x(1)} = x(2), lim\_inf_{x(2)} = x(1), lim\_sup_{x(3)} = x(4), \cdots, lim\_inf_{x(L)} = x(L-1)$, $lim\_sup_{x(L)} = lim\_sup$.
7) Calculate the fitness values associated with each offspring $(\mathbf{x}'_i, \sigma'_i, \forall i \in \{1, \cdots, \mu\})$.
8) Conduct pairwise comparison over the union of parents and offspring: For each individual, p opponents are chosen uniformly at random from all the parents and offspring. For each comparison, if the individual's fitness is better than the opponent's, it receives a "win."
9) Select the μ individuals out of the union of parents and offspring that have the most wins to be parents of the next generation.
10) Stop if the halting criterion is satisfied, and if not, set $k = k + 1$ and go to Step 3.

VI. Computational Experiments and Results

In this section, we present several examples in order to illustrate the computational performance of the proposed algorithms, and to compare their convergence and robustness with the previous approaches. All the minimization algorithms have been implemented using MATLAB.

A. Modified Discrete Fourier Transform (MDFT) FBs

A scheme of modulation by which all the analysis and synthesis (or the receiving and the transmitting) subchannel filters can be obtained for different families of MFBs (or MCMs) can be defined in the following way. Let $P(z) = \sum_{n=0}^{N-1} p[n] \cdot z^{-n}$ be the system function of the N-length real coefficients prototype filter $p[n]$ appropriately designed. We define $S_i(z)$, $0 \leq i \leq 2M-1$, as follows:

$$S_i(z) = a_i \cdot P\left(zW_{2M}^{(i+\gamma)}\right) + b_i \cdot P\left(zW_{2M}^{-(i+\gamma)}\right) \tag{18}$$

where γ is an arbitrary modulation factor, and $H_i(z)$ and $F_i(z)$ are subchannel filters obtained as

$$H_i(z) = \begin{cases} S_i(z) & i \text{ is even} \\ z^{-(N-1)} \cdot \tilde{S}_i(z) & i \text{ is odd} \end{cases} \tag{19a}$$

$$F_i(z) = c_i \cdot z^{-(N-1)} \cdot \tilde{H}_i(z) \tag{19b}$$

where $\tilde{S}_i(z) = S_i(z^{-1})$. If we apply the inverse z-transform, we obtain the following expressions:

$$h_i[n] = \begin{cases} s_i[n] & i \text{ is even} \\ s_i[N-1-n] & i \text{ is odd} \end{cases} \tag{20a}$$

$$f_i[n] = c_i \cdot h_i[N-1-n]. \tag{20b}$$

From the filters above, we can obtain several kinds of MFBs, and, therefore, different MCMs. In [9], a means for obtaining NPR CMFBs from the proposed modulation scheme is shown (considering $\gamma = 1/2$). Another example is as follows. We can obtain the Type-1 MDFT filter banks [39] by considering the following parameters:

$$a_i = \begin{cases} W_{2M}^{i\cdot(N-1)/2} & i \text{ is even} \\ W_{2M}^{-i\cdot(N-1)/2} & i \text{ is odd} \end{cases} \qquad b_i = 0$$

$$\gamma = 0, \quad c_i = M \cdot W_{2M}^{i\cdot(N-1)}. \tag{21}$$

By algebraic manipulation we obtain

$$h_i[n] = (1/M) \cdot f_i[n] = p[n] \cdot W_{2M}^{-i\cdot(n-(N-1)/2)}. \tag{22}$$

TABLE II
QUALITY MEASURES IN 32-CHANNEL MDFT FBs. THE 513-LENGTH PROTOTYPE FILTERS HAVE BEEN OPTIMIZED BY MEANS OF DIFFERENT ALGORITHMS

Nelder-Mead (NM)-based optimization

	$L = 3$ (samples in the transition band)		$L = 4$ (samples in the transition band)	
Quality Parameter	$f(k)$ as (11)	$f(k)$ as (12)	$f(k)$ as (11)	$f(k)$ as (12)
ψ_{min}	2.5×10^{-3}	2.138×10^{-4}	4.2448×10^{-3}	9.6904×10^{-4}
MSA (dB)	41.8	57.9	38.5	47.6
R_{pp}	3.2605×10^{-2}	3.203×10^{-3}	6.1712×10^{-2}	1.4189×10^{-2}
E_a (dB)	-43.9172	-58.9811	-40.4752	-49.4939
PSNR (dB)	46.1706	63.0940	41.5751	53.0289

quasi-Newton (qN)-based optimization

	$L = 3$ (samples in the transition band)		$L = 4$ (samples in the transition band)	
Quality Parameter	$f(k)$ as (11)	$f(k)$ as (12)	$f(k)$ as (11)	$f(k)$ as (12)
ψ_{min}	2.1731×10^{-4}	3.1966×10^{-4}	2.3978×10^{-3}	2.0088×10^{-4}
MSA (dB)	56.3	72.1	63.7	58.2
R_{pp}	3.2749×10^{-3}	4.9159×10^{-3}	2.4212×10^{-2}	3.6232×10^{-3}
E_a (dB)	-57.6232	-89.5441	-65.5341	-59.3831
PSNR (dB)	62.5679	66.1573	49.2529	63.5662

Proposed Evolutionary Programming Approaches

	$L = 3$ (samples in the transition band)		$L = 4$ (samples in the transition band)	
Quality Parameter	Proposed VLFEP	Proposed VLCEP	Proposed VLFEP	Proposed VLCEP
ψ_{min}	2.1233×10^{-4}	2.1231×10^{-4}	1.9854×10^{-4}	1.9947×10^{-4}
MSA (dB)	58	58	59	58.6
R_{pp}	3.277×10^{-3}	3.2769×10^{-3}	3.6394×10^{-3}	3.718×10^{-3}
E_a (dB)	-59.0421	-59.047	-60.0017	-59.6475
PSNR (dB)	63.0498	63.051	63.732	63.5762

B. Results

For ease of comparison, we give examples with the same design parameters, as follows:

- The cost function to be minimized is that of given in (9). It has been selected after designing the same examples given below with other ψ functions (results not shown here), with this function giving consistently the best results.
- $\alpha = 0$ in (5).

In order to evaluate the quality of the resulting filter banks, we measure the minimum obtained in the optimized cost function ψ and the minimum stopband attenuation (MSA) calculated from the prototype filters. We also give the values of the peak amplitude distortion R_{pp} and the maximum aliasing error E_a introduced by the FBs, defined as in [2] in [40]. Finally, we present the peak signal-to-noise ratio (PSNR) obtained after applying a sequence of random numbers uniformly distributed between 1 and -1 to the transmitter-receiver combination. For a Perfect Reconstruction FB, this PSNR would be infinite (not accounting for numerical quantization effects), and for our NPR FBs, this gives a measure of noise related to the filter bank design only (the reconstruction noise). With regard to the convergence time of the proposed approaches in order to obtain optimal values of the samples, their computational complexity has been compared experimentally. Specifically, we have compared the run-times of nonoptimized versions of the algorithms using the MATLAB programming language. All the simulations were carried out on a Pentium 4 CPU clocked at 3.00 GHz, with 1 GB of installed physical memory. In all the simulations, the VL-CEP and VL-FEP algorithms were run with a population of 100 individuals, for 100 generations.

First, we consider 32-channel MDFT filter banks, where all analysis and synthesis filters are obtained as given in (22). The prototype filters have been designed starting from (11) or (12) for the transition band samples, and optimized by means of Nelder-Mead (NM) or quasi-Newton (qN) algorithms through `fminsearch` and `fminunc` functions, both included in the Optimization Toolbox of MATLAB. We also employ the proposed EP approaches, and all the filters obtained have a length of 513 coefficients. This length has been chosen in order to implement the MDFT filter bank by means of a fast algorithm [40]. In this case, we run the EP algorithms proposed 30 times, selecting the magnitude values of the stopband given by the minimum cost function value ψ obtained.

Table II shows the results obtained after fixing the number of the samples in the transition band to $L = 3$ and $L = 4$, respectively, by considering the qN and NM algorithms starting from

TABLE III
OPTIMAL RESULTS FOR 32-CHANNEL MDFT FBs (513-LENGTH PROTOTYPE FILTERS), WHERE THE NUMBER OF SAMPLES IN THE TRANSITION BAND IS VARIABLE

	$f(k)$ as (11) NM optimization	$f(k)$ as (12) NM optimization	$f(k)$ as (11) qN optimization	$f(k)$ as (12) qN optimization	Proposed VLFEP	Proposed VL-CEP
L	7	7	9	7	6	5
ψ_{min}	9.3048×10^{-5}	3.7508×10^{-5}	4.7658×10^{-5}	1.4834×10^{-4}	1.891×10^{-5}	3.0111×10^{-5}
MSA (dB)	61.3	83.5	66.2	79.3	69.3	74.9
R_{pp}	9.6231×10^{-4}	4.2506×10^{-4}	6.0983×10^{-4}	1.8589×10^{-3}	3.1296×10^{-4}	4.6274×10^{-4}
E_a (dB)	-67.0746	-84.2963	-77.8974	-76.2632	-73.2686	-85.5957
PSNR (dB)	73.5754	84.0786	78.85	72.762	78.9039	85.1304
Computational Time (seconds)	4.2	3.0	7.1	4.3	61.4	61.4

TABLE IV
OPTIMAL RESULTS FOR 128-CHANNEL MDFT FBs (2049-LENGTH PROTOTYPE FILTERS), WHERE THE NUMBER OF SAMPLES IN THE TRANSITION BAND IS VARIABLE

	$f(k)$ as (11) NM optimization	$f(k)$ as (12) NM optimization	$f(k)$ as (11) qN optimization	$f(k)$ as (12) qN optimization	Proposed VLFEP	Proposed VL-CEP
L	5	7	9	7	5	7
ψ_{min}	1.1552×10^{-4}	4.9344×10^{-5}	1.5953×10^{-5}	5.679×10^{-5}	1.1075×10^{-5}	2.7426×10^{-5}
MSA (dB)	88.8	77.4	64.2	73.6	69	65.1
R_{pp}	1.6753×10^{-3}	7.0193×10^{-4}	1.594×10^{-4}	1.2488×10^{-3}	1.2848×10^{-4}	4.7509×10^{-4}
E_a (dB)	-114.11	-114.85	-98.0062	-108.43	-93.2736	-88.1592
PSNR (dB)	74.1764	79.3911	83.9646	77.1304	84.9108	78.3094
Computational Time (seconds)	10.7	23.3	58.7	25.4	310.5	310.5

different initial values for $f(k)$ [(11) or (12)], and also using the proposed EP algorithms (best solution obtained). Finally, the complete set of best-case results are shown in Table III. In this case, the number of samples in the transition band has not been fixed, and, hence, varies from case to case. For visualization purposes, the magnitude responses of the optimized prototype filters are plotted in Fig. 1. Clearly, the filter banks designed with the proposed techniques can be considered to very nearly satisfy the PR property, since the amplitude distortion functions are approximately flat (with peak normalized deviations of only 3.1296×10^{-4} and 4.6274×10^{-4}), and aliasing error is almost suppressed (worst aliasing peaks are -73.2686 and -85.5957 dBs). Our results also give a good illustration of the utility of the objective function. Although the proposed EP algorithms always obtain better values for the optimized cost function (1.891×10^{-5} and 3.0111×10^{-5}) than the other tested approaches, they do not always provide all the best quality parameters. For example, the best case for MSA (83.5 dBs) is obtained using the Nelder-Mead approach. However, in general, finding a good value for the objective function (as guaranteed by the EP approaches) usually corresponds to suitable values for the other metrics.

In our second example, we increase the number of channels and the filter lengths for the MDFT FBs to 128 and 2049, respectively. The resulting systems can be applied for designing MCMs for data transmission. Table IV shows the measures corresponding to the complete set of proposed systems. Fig. 2 depicts the magnitude response of the prototype filters optimized by means of VLFEP and VLCEP algorithms. Note that for the proposed approaches, the amplitude distortion peaks are 1.28486×10^{-4} and 4.7509×10^{-4}, and the worst aliasing peaks are -93.2736 and -88.1592 dBs. As in the earlier example, the best optimized value for the objective cost has also been obtained with the proposed VLFEP algorithm: 1.1075×10^{-5}. Again, however, that does not always guarantee the best set of quality parameters. For this reason, ongoing research can be aimed at establishing formal links between the objective function to be minimized and the quality parameters considered, allowing the proposal of a new ψ functions that guarantees all the best results.

Regarding the convergence effort of the proposed algorithms, although the computational time is increased compared with qN and NM-based approaches (Tables III and IV), the amount of time required by the VL-CEP and VL-FEP algorithms is still acceptable (5 min in the worse case for the 128 channels, 2049-length prototype filter). In order to perform simulations with similar amount of computational time for all the algorithms tested, we have incorporated a local procedure (hill-climbing [48]) to be run after the NM and qN algorithms. The hill-climbing local optimizer is repeatedly run until the

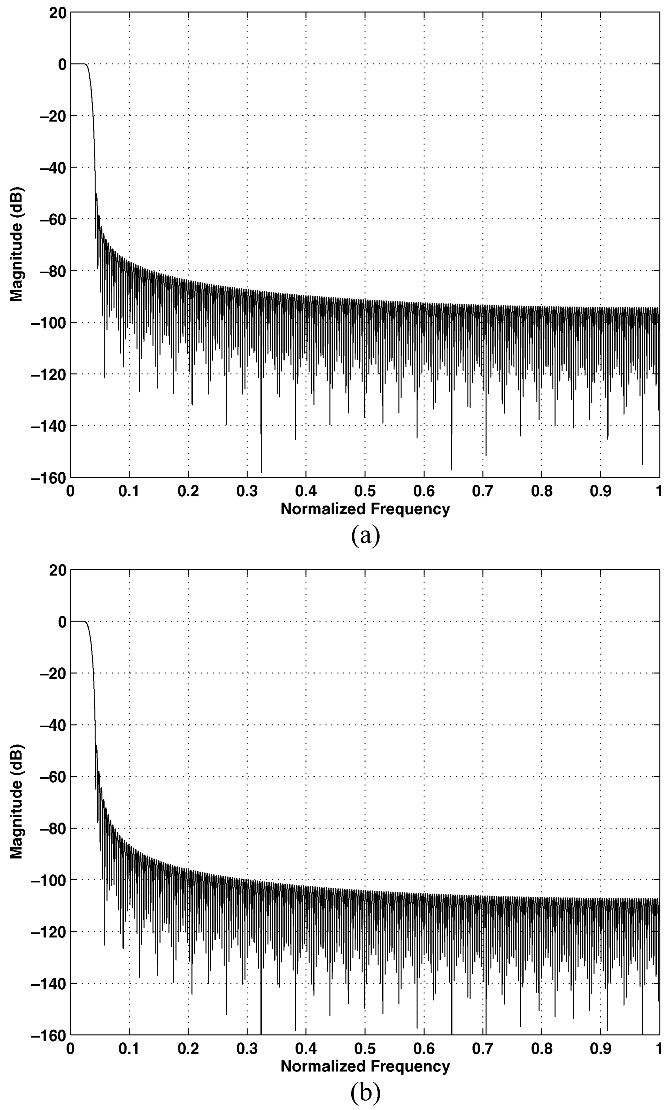

Fig. 1. Magnitude response plot of optimized: (a) VLFEP and (b) VLCEP prototype filters for 32-channel MDFT FB.

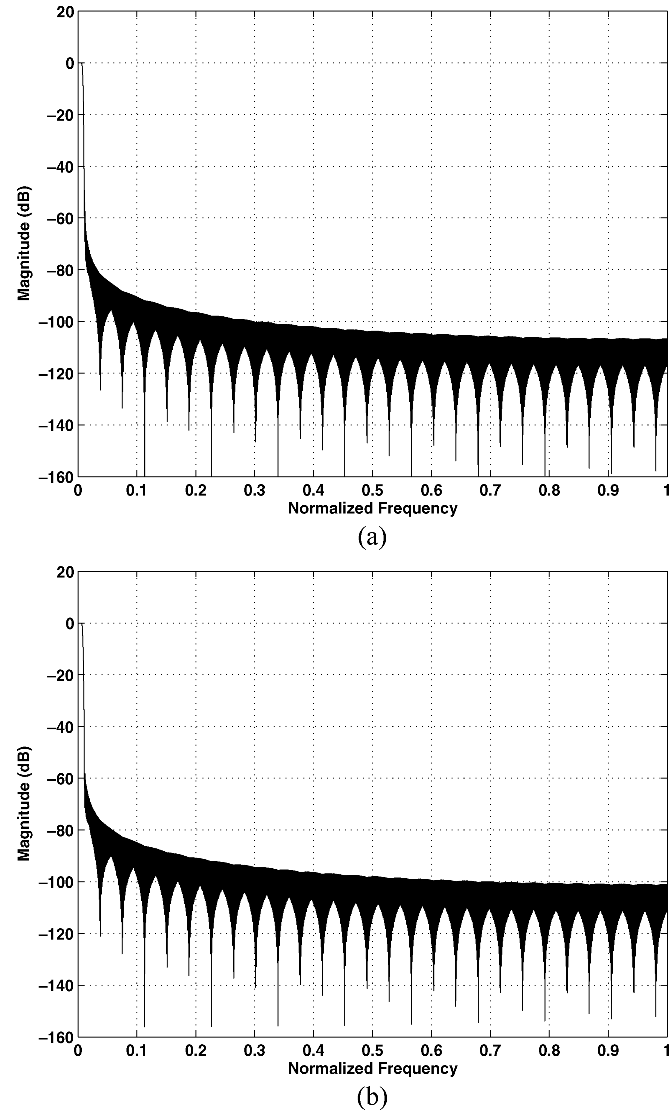

Fig. 2. Magnitude response plot of optimized: (a) VLFEP and (b) VLCEP prototype filters for 128-channel MDFT FB.

same processing time of the Evolutionary Programming approaches is used. An outline of the hill-climbing algorithm is the following:

Hill-climbing Local Search Procedure

Let $\mathbf{x}$ be an initial solution;

for $(i = 1 : Max\_local)$

  $\mathbf{x}' \leftarrow \mathbf{x}$

  Mutate $(\mathbf{x}')$

  Compute $\psi(\mathbf{x}')$;

    if $(\psi(\mathbf{x}') < \psi(\mathbf{x}))$

      $\mathbf{x} \leftarrow \mathbf{x}'$

    endif

endfor

TABLE V
RESULTS OBTAINED FOR THE 32-CHANNEL MDFT FBS (513-LENGTH PROTOTYPE FILTERS) USING NM OPTIMIZATION MIXED WITH A HILL-CLIMBING LOCAL ALGORITHM

	$f(k)$ as (12), NM with hillclimbing
L	7
ψ_{min}	3.7195×10^{-5}
MSA (dB)	87.3
R_{pp}	3.9811×10^{-4}
E_a (dB)	-108.0435
PSNR (dB)	84.5609
Computational Time (seconds)	62.8

Note that the variable Max_local controls the number of function evaluations, and therefore the computational time of

TABLE VI
AVERAGE AND STANDARD DEVIATION VALUES (AVG/STD. DEV) OBTAINED AFTER 30 RUNS OF THE COMPARED ALGORITHMS WITH RANDOM INITIALIZATION, IN THE TWO EXAMPLES PROPOSED 32-CHANNEL AND 128-CHANNEL MDFT FILTER BANKS

32-channel				
L	proposed VLCEP ($\times 10^{-3}$)	proposed VLFEP ($\times 10^{-3}$)	NM optimization ($\times 10^{-3}$)	qN optimization ($\times 10^{-3}$)
3	0.26/0.17	0.32/0.25	7.1/8.9	9.0/8.6
4	0.28/0.19	0.35/0.31	2.8/4.5	9.5/16.9
5	0.27/0.23	0.40/0.28	6.7/19.6	18.5/1.3
6	0.22/0.18	0.43/0.28	3.5/53.2	8.2/9.6
7	0.56/0.51	0.47/0.57	37.3/46.3	22.6/24.5
8	0.45/0.37	0.33/0.23	99.4/71.5	16.4/21.0
128-channel				
L	proposed VLCEP ($\times 10^{-3}$)	proposed VLFEP ($\times 10^{-3}$)	NM optimization ($\times 10^{-3}$)	qN optimization ($\times 10^{-3}$)
3	0.17/0.15	0.22/0.11	8.2/8.8	8.1/8.9
4	0.21/0.19	0.34/0.24	3.2/6.5	17.5/1.1
5	0.32/0.25	0.33/0.28	14.7/23.1	14.7/18.0
6	0.44/0.52	0.37/0.25	12.3/25.9	12.0/19.8
7	0.37/0.43	0.28/0.19	11.3/13.6	12.0/11.9
8	0.39/0.41	0.43/0.47	103.4/85.4	20.1/28.8

TABLE VII
t VALUES OBTAINED BY A TWO-TAILED T-TEST FOR THE 32-CHANNELS AND 128-CHANNELS MDFT FILTER BANKS EXAMPLES. † STANDS FOR VALUES OF t WITH 29 DEGREES OF FREEDOM WHICH ARE SIGNIFICANT AT $\alpha = 0.05$

32-channel				
L	VLCEP-NM	VLFEP-NM	VLFCEP-qN	VLFEP-qN
3	$-3.27^{\dagger}$	$-3.01^{\dagger}$	$-3.87^{\dagger}$	$-3.91^{\dagger}$
4	-2.1	-2.19	-2.1	-2.13
5	-1.24	-1.27	$-4.41^{\dagger}$	$-4.61^{\dagger}$
6	$-2.54^{\dagger}$	$-2.55^{\dagger}$	$-3.12^{\dagger}$	$-3.22^{\dagger}$
7	$-3.65^{\dagger}$	$-3.64^{\dagger}$	$-3.49^{\dagger}$	$-3.44^{\dagger}$
8	$-5.36^{\dagger}$	$-5.33^{\dagger}$	$-2.96^{\dagger}$	$-2.95^{\dagger}$
128-channel				
L	VLCEP-NM	VLFEP-NM	VLFCEP-qN	VLFEP-qN
3	$-3.21^{\dagger}$	$-3.37^{\dagger}$	$-3.39^{\dagger}$	$-3.37^{\dagger}$
4	-1.81	-1.80	$-4.10^{\dagger}$	$-4.01^{\dagger}$
5	-1.59	-1.67	-2.08	$-2.22^{\dagger}$
6	-2.01	-2.11	$-2.42^{\dagger}$	$-2.31^{\dagger}$
7	$-3.25^{\dagger}$	$-3.19^{\dagger}$	$-3.68^{\dagger}$	$-3.73^{\dagger}$
8	$-6.50^{\dagger}$	$-6.43^{\dagger}$	$-2.70^{\dagger}$	$-2.63^{\dagger}$

the local algorithm. Using this approach, we have only obtained an improvement in the design of one filter, specifically in the design of the 32-Channel MDFT FBs (513-length Prototype Filters), using the initial solution given by the NM algorithm starting from (12). The parameters of the new filter are given in Table V. Note also that the new filter improves some quality parameters such as MSA and E_a, but it shows worse $PSNR$ and R_{pp} values than the filters designed with the evolutionary techniques. In addition the objective function found by the evolutionary approaches proposed in this paper is better than the one found by the NM with hill-climbing. In the rest of the tests shown in Tables III and IV we did not obtain any improvement in the filters after running the local search procedure.

We have shown that the optimization algorithms compared in this paper provides good prototype filters. In order to complete our study, it is important to test if the compared algorithms are sensitive to the initial magnitude values for the transition band samples. We have run the VLCEP, VLFEP, NM and qN optimization algorithms in the two examples that we consider, the 32-channel and 128-channel MDFT filter banks. Table VI shows the values of the average and standard deviation obtained after 30 runs of the algorithms. Note that the results obtained by the VLCEP and VLFEP are consistently better than the results provided by the NM and qN methods initialized with random magnitude values in the transition band. Table VII shows the results of a t-test performed over the data obtained by the compared algorithms. It is apparent that our approaches VLCEP and VLFEP, perform statistically equally or better than NM and qN algorithms in all the experiments performed.

VII. CONCLUSION

Two EP algorithms are proposed which allow robust minimization of cost functions which are applicable to the design of filters using a frequency-sampling technique. By minimizing these cost functions, we have obtained prototype filters with desirable characteristics to use them in modulated filter banks, namely, high stopband attenuation, minimal reconstruction noise, and minimal in-band amplitude distortion. A key advantage of the proposed algorithms over previous design approaches is that they are not sensitive to the choice of initial conditions at algorithm start-up.

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