Lower bound on number of ADMs in WDM rings with nonuniform traffic demands

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A new and tight lower bound on the number of ADMs with arbitrary nonuniform traffic demands in a SONET/WDM ring network is derived. Simulations show that this lower bound is much tighter than the previous one and for some cases it reaches the infimum.

Introduction: In traffic grooming, the lower bounds on the numbers of ADMs and wavelengths are important parameters in evaluating the grooming results. Although much work has been done on the grooming of traffic in WDM ring networks and the lower and upper bounds on the number of wavelengths and the upper bound on that of ADMs are relatively easy to derive, the lower bound on the number of ADMs has only been derived for uniform traffic demands [1–3] or in some special cases for nonuniform ones, in which the traffic demand was assumed to be a unit, i.e. either 0 or 1 [1, 2]. We have derived a lower bound on the number of ADMs with arbitrary nonuniform traffic demands [4]. However, that lower bound is not tight enough as to effectively evaluate the grooming results. In this Letter, a much tighter lower bound with nonuniform traffic demands is derived. In some cases, this new lower bound reaches the infimum.

Unidirectional rings: The lower bound on the number of ADMs can be reached by one of the following two approaches. 1. If the total traffic sourcing from or terminating at each node is computed, the number of ADMs must be able to support this amount of traffic at each node. 2. If the number of nodes each wavelength drops at is known, the minimum number of ADMs is the sum of the nodes each wavelength drops at.

The first approach was discussed in detail in [4] and the lower bound obtained from it is denoted as $M_{\min 1}$ in this Letter.

For the second approach, to reach the theoretical lower bound, the minimum number of wavelengths $W_{\min}$ must be used to support the traffic demands on the ring, which was obtained in [4], and is the infimum of the number of wavelengths. If an all-to-all traffic scheme is considered, the average number of traffics each wavelength carries is $n(n-1)/W_{\min}$. However, if the wth wavelength drops at μ_w nodes, it can at most carry $\mu_w(\mu_w-1)$ connections. To minimise the number of ADMs, the average number of traffics each of the $W_{\min}$ wavelengths carries must satisfy

$$E[\mu_w(\mu_w-1)] \geq \frac{n(n-1)}{W_{\min}} \tag{1}$$

where $E[x]$ is the average of x.

(1) has to be subjected to the following adaptations. (i) *Partial traffic adaptation*: If there are only partial traffic demands on the ring, the total number of traffics in (1) must be replaced by the number of nonzero traffic demands N. (ii) *Unpairment adaptation*: If some of the traffic demands are unpaired, i.e. when $t_{ij}=0$, $t_{ji}\neq 0$, where t_{ij} represents the traffic demand from node i to node j, a wavelength dropping at μ_w nodes cannot carry $\mu_w(\mu_w-1)$ traffics. To take into account of this situation, we define the unpairment of a traffic pattern as

$$\eta = \frac{\text{(number of unpaired traffics)}}{\text{(number of nonzero traffics)}} \tag{2}$$

If $\eta=0$, all the traffic demands are fully paired and no amendment is needed. If $\eta=1$, the traffic is totally unpaired – only one traffic demand between each pair of nodes is nonzero. In this case, each wavelength can at most carry $\mu_w(\mu_w-1)/2$ traffics when it drops at μ_w nodes.

Taking into consideration the above two effects, (1) becomes:

$$E[\mu_w(\mu_w-1)] \geq \frac{2N_p}{W_{\min}} \tag{3}$$

where $N_p=(\eta+1)N$ is the total number of traffic pairs, including the unpaired ones.

If the number of nodes each wavelength drops at does not differ much from each other, the left side of (3) can be simplified as

$$E[\mu_w(\mu_w-1)] \simeq E[E[\mu_w]\mu_w-\mu_w] = \bar{\mu}(\bar{\mu}-1) \tag{4}$$

where $\bar{\mu}$ is the average number of nodes each wavelength drops at. Therefore, the lower bound on the number of ADMs is obtained

$$M_{\min 2} = \left\lfloor \frac{W_{\min}+\sqrt{W_{\min}^2+8W_{\min}N_p}}{2} \right\rfloor \tag{5}$$

The tighter lower bound is the larger one of the above two

$$M_{\min} = \max\{M_{\min}, M_{\min 2}\} \tag{6}$$

Generally, $M_{\min 2} > M_{\min 1}$, thus the tighter lower bound on the number of ADMs is $M_{\min}=M_{\min 2}$.

Bidirectional rings: For bidirectional rings, we use the deterministic shortest path routing (DSPR) to assign a fixed shortest path route for each traffic. For n odd, there is no ambiguity, but for n even, there are two shortest paths from node i to node $i\oplus(n/2)$ (the symbol $\oplus$ denotes the addition and $\ominus$ the subtraction of modulo-n). In the later case, the traffic between nodes i and $i\oplus(n/2)$ is routed clockwise if i is even and counterclockwise if i is odd. In this way, the rings in the two directions are antisymmetric and an expression derived in one direction can be used in the other by simply substituting $n-1-i$ for i and $n-1-j$ for j in the subscript indices. Therefore we derive only the expressions in the clockwise direction in the discussions below.

Similar to the unidirectional case, the lower bound on the number of ADMs can also be reached by the two approaches described before. If we use $\sigma(i)$ to denote the egress load, the total traffic sourcing from node i, and $\tau(i)$ the ingress load, that terminating at node i, of node i in one direction, using the derivative process similar to the one given in [4], the first lower bound on the number of ADMs can be obtained by

$$M_{\min 1} = \sum_{i=0}^{n-1}\left\lfloor \left\lceil \frac{Max(\sigma(i),\tau(i))}{g} \right\rceil + \left\lceil \frac{Max(\sigma(n-1-i),\tau(n-1-i))}{g} \right\rceil \right\rfloor \tag{7}$$

where g is the traffic granularity. The first term represents the minimum number of ADMs in the clockwise direction and the second in the counterclockwise direction. Furthermore, $\sigma(i)$ and $\tau(i)$ can be computed by

$$\sigma(i) = \sum_{l=i\oplus 1}^{i\oplus\Delta(i)} t_{il} \tag{8}$$

and

$$\tau(i) = \sum_{k=i\ominus\Delta(i+n/2)}^{i\ominus 1} t_{ki} \tag{9}$$

respectively, where

$$\Delta(x) = \begin{cases} (n-1)/2, & \text{if } n \text{ is odd} \\ n/2 - x \bmod (2), & \text{if } n \text{ is even} \end{cases} \tag{10}$$

In (8), the summation indicates that the traffic from node i can reach only those nodes which are on the downstream of it and at a distance no farther than $\Delta(i)$ from it in the clockwise direction. The summation in (9) has the similar meaning. In addition, these two summations are carried out around the ring, that is to say when the index exceeds $n-1$ but does not reach its upper bound, the summation proceeds from node 0 to its upper bound.

According to the second approach, since there are only $n(n-1)/2$ traffic demands in each direction, the average number of traffic demands each wavelength carries in any direction is $n(n-1)/2W_{\min}^d$, where $W_{\min}^d$ is the minimum number of wavelengths in direction d, $d=0$, 1 representing the clockwise and counterclockwise directions, respectively.

$W_{\min}^d$ can be obtained by the derivation similar to the one given in [4] with the link load L_i from node i to node $i\oplus 1$ in the clockwise direction is given by

$$L_i = \sum_{k=i+1 \ominus \Delta(i+1+n/2)}^{i} \sum_{l=i \oplus 1}^{k \oplus \Delta(k)} t_{kl} \tag{11}$$

where $\Delta(x)$ is given by (10).

In (11), the outer summation indicates that only those traffics whose source nodes are at the upstream of node $i \oplus 1$ and at a distance no farther than $\Delta(i+1+n/2)$ from that node can traverse link L_i. The inner summation denotes that the traffics sourcing from node k can traverse this link if their destination nodes are at or in the downstream of node $i \oplus 1$ but at a distance no farther than $\Delta(k)$ from node k. Again, these two summations are also carried out around the ring until the upper bound is reached. The maximum link load $L_{\max}^{d}$ is the maximum of the link load in direction d.

Note that in the bidirectional case, a wavelength dropping at μ nodes can at most accommodate $\mu(\mu-1)/2$ traffic demands. By employing the similar derivation process adopted in unidirectional rings, the second lower bound on the number of ADMs is given by

$$M_{\min 2} = \sum_{d=0}^{1} \left\lfloor \frac{W_{\min}^{d} + \sqrt{(W_{\min}^{d})^2 + 4W_{\min}^{d} N^{d}}}{2} \right\rfloor \tag{12}$$

where N^{d} is the number of nonzero traffics in direction d.

In (12) we do not take into account the unpaired adaptation since the unpairment does not always affect the number of traffics each wavelength carries in the bidirectional case. Consider, for example, a five-node ring with traffic granularity $g=4$ OC-3's. The traffic demands $0 \to 2$, $2 \to 3$, $3 \to 0$, and $0 \to 4$, $4 \to 2$, $2 \to 0$ are all 4 OC-3's. Although four of the six traffics are unpaired, the wavelength in the clockwise direction dropping at nodes 0, 2, and 3 can carry the first three traffic demands and that in the counterclockwise direction dropping at nodes 0, 2, and 4 can also carry the remaining three traffics. Therefore, by adjusting the dropping nodes of each wavelength, it can accommodate $\mu(\mu-1)/2$ traffic demands when dropping at μ nodes even though some of the traffics are unpaired. But this is not always guaranteed so that the lower bound in this case is not always as tight as in unidirectional rings.

The tighter lower bound one is the greater of the two

$$M_{\min} = \max\{M_{\min 1}, M_{\min 2}\} \tag{13}$$

Simulations: We will give two examples to show the performance of the new lower bound derived in this Letter. We first consider the grooming problem with the traffic pattern given in [1] on a unidirectional ring with eight nodes: $T = \{(0,5), (5,2), (2,7), (7,4), (4,1), (1,6), (6,3), (3,0)\}$, the traffic granularity is the same as the traffic demand. After splitting (cutting) some of the traffics into two segments (denoted by bold letters), the traffic pattern becomes: $T' = \{(0,5), \mathbf{(5,0), (0,2)}, (2,7), \mathbf{(7,0), (0,4)}, \mathbf{(4,0), (0,1)}, (1,6), \mathbf{(6,0), (0,3)}, (3,0)\}$. Hence, traffic T and T' are exactly the same. The optimal assignment of those two patterns to wavelengths results in five wavelengths and 12 ADMs. From (5), the lower bounds on the number of ADMs calculated for the first and the second traffic patterns are 11 and 12, respectively. We see that one of the lower bounds reaches its infimum.

Table 1 gives the two lower bounds obtained from [4] ($M_{\min 1}$) and this Letter ($M_{\min 2}$) for rings with eight to 16 nodes and different traffic granularities. The traffic demands are the random integers between [0, 15]. We find that $M_{\min 2}$ is always tighter than $M_{\min 1}$ in all the cases. The larger the traffic granularity is, i.e. the lighter the traffic load is, the greater the difference between these two bounds is. When $g=96$, $M_{\min 2}$ is nearly twice of $M_{\min 1}$. We also found from our simulations with the algorithm given in [5] that $M_{\min 2}$ sometimes reached its infimum for large g's or light traffic demands.

Table 1: Comparison of the two lower bounds on number of ADMs in unidirectional rings

Number of nodes		8	9	10	11	12	13	14	15	16
$g=16$	$M_{\min 1}$	32	43	54	58	75	87	98	114	132
	$M_{\min 2}$	38	47	67	69	86	99	114	132	156
$g=24$	$M_{\min 1}$	24	28	39	43	52	61	66	81	92
	$M_{\min 2}$	30	37	48	54	67	78	90	103	123
$g=48$	$M_{\min 1}$	15	18	20	24	28	35	37	45	49
	$M_{\min 2}$	21	26	33	36	46	52	60	68	82
$g=96$	$M_{\min 1}$	8	9	10	13	16	22	23	30	31
	$M_{\min 2}$	14	19	23	26	31	36	42	48	57

Conclusion: We have derived a tighter lower bound on the number of ADMs for both unidirectional and bidirectional rings with arbitrary traffic demands in this Letter. Numerical results showed that the new lower bound is much tighter than the previous one, and in some cases, it reaches the infimum of the problem especially for the cases in which the traffic demands are far less than the wavelength's capacity.

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