Artificial Intelligence and Natural Computation seminar
2 Sep 2019
School of Computer Science
http://talks.bham.ac.uk/talk/index/3844

Why current AI and neuroscience fail to replicate or explain ancient forms of spatial reasoning and mathematical consciousness

Aaron Sloman
Honorary Professor of AI and Cognitive Science
School of Computer Science
http://www.cs.bham.ac.uk/~axs

3D spatial reasoning in a pre-verbal toddler (about 17.5 months):
http://www.cs.bham.ac.uk/research/projects/cogaff/movies/pencil.webm

---

David Hume and Immanuel Kant (from Wikimedia)

Very crudely, David Hume, depicted above, on the left, claimed that there are only two kinds of knowledge:

1. empirical knowledge that comes through sensory mechanisms, possibly aided by measuring devices of various kinds; which he called knowledge of "matters of fact and real existence". Deep learning mechanisms can achieve this.
2. what he called "relations of ideas", which we can think of as things that are true by definition, such as "All bachelors are unmarried", and (if I've understood Hume rightly) all mathematical knowledge, for example ancient knowledge of arithmetic and geometry, which Hume seemed to suggest was no more informative than the bachelor example.

"True by definition" applies to all truths that can be proved using only logic and definitions. An example is "No bachelor uncle is an only child", which can easily be proved from the definitions of "bachelor", "uncle" and "only child", using only logical reasoning.

Hume famously claimed that if someone claims to know something that is neither of type 1 (empirical) nor of type 2 (mere definitional truths) we should "Commit it then to the flames: for it can contain nothing but sophistry and illusion", which would have included much philosophical writing by metaphysicians, and theological writing.

Immanuel Kant's response (1781)

In response to Hume, Immanuel Kant, depicted above, on the right, claimed that there are some important kinds of knowledge that don't fit into either pair of Hume's two categories ("Hume's fork"), for they are not mere matters of definition, or derivable from definitions purely by using logic, i.e.

- they are not analytic but synthetic, e.g. not provable simply using definitions and logic, and
- they also are not based on experience in such a way that future experiences might prove them false, as could happen with most supposed knowledge about the world around us: i.e. they are not empirical.
- If they are true there are no possible circumstances in which they could be false, i.e. they are necessarily true, not contingently true.
  Likewise if they are false, e.g. **3 + 5 = 9**, then there are no possible circumstances in which they could be true, i.e. they are necessarily false, not contingently false.

Example of an empirical, contingent, synthetic proposition: The fact that for millions of years sunset at any location on this planet has always been followed by sunrise does not prove that that will always be the case, or that it is necessarily the case, since, for example, after some future sunset a huge asteroid might collide with the earth, smashing it to pieces and preventing any future sunrise here.

In his argument against Hume, Kant drew attention to kinds of mathematical knowledge that do not fit into either of Hume's two categories: since we can discover by means of special kinds of non-logical, non-empirical reasoning (that he thought was deeply mysterious, since he was unable to explain it), that "5+3=8" is a necessary truth, but not a mere matter of definition, nor derivable from definitions using only logic. (Unlike most philosophers I think such propositions are ambiguous and in one interpretation they conform to Kant's theories, but not the interpretation Hume gave them.)

Kant thought such mathematical discoveries in arithmetic, and discoveries in Euclidean geometry were synthetic, not analytic and also could not possibly be false, so they are necessary truths, and because they are not based on or subject to refutation by observations of how things are in the world, such knowledge is non-empirical, i.e. a priori.

For a more careful and detailed, but fairly brief explanation of Kant's three distinctions. apriori/empirical, analytic/synthetic and necessary/contingent, see
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/kant-maths.html

My 1962 DPhil thesis was an attempt to defend Kant against critics, such as Carl G. Hempel who thought Kant had been proved wrong

- because work by Frege, Russell and others had shown that arithmetical knowledge was reducible to logic, and therefore analytic;
- because Kant's claim that Euclid's geometrical axioms and theorems were necessarily true of the nature of space had been proven false by Einstein's general theory of relativity, supported by evidence gained by Eddington and collaborators during the 1919 eclipse of the sun as argued by Hempel (ref above), and others:
  https://en.wikipedia.org/wiki/Solar_eclipse_of_May_29,_1919

It is widely, but erroneously, believed that Immanuel Kant's philosophy of mathematics in his Critique of Pure Reason (1781) was disproved by Einstein's theory of general relativity (confirmed by Eddington's observations of the solar eclipse in 1919, establishing that physical space is non-Euclidean).

That belief is erroneous if Kant was not making a claim about physical space but about one of the types of space that we can think about, e.g. by imagining some basic features, or abstracting them from perceived objects (e.g. the 2D space on the surface of a spherical object, or an egg shaped object, or a toroidal -- circular tube shaped -- object), and then deriving implications of these basic features, by thinking about the features themselves, i.e. not merely manipulating sentences describing those features.

Without much difficulty you should be able, for example, to think of alternatives to a circular tube forming a 3D ring, or toroid, by imagining various deformations of that shape, e.g. twisting it into a figure 8-like shape, or introducing sharp corners and flat surfaces, turning the type in a square picture-frame like shape, perhaps with with a very thick frame. Some of the mathematically possible deviations from familiar Euclidean space are much harder to think about than others, e.g. thinking about 1000-dimensional shapes embedded in a 1001-dimensional space.

I suggest that just as we can use sentences in a spoken, written, or thought language to consider new possibilities and then derive consequences of those possibilities, we can also use non-linguistic forms of representation to visualise possibilities and then derive consequences. That's the sort of thing ancient mathematicians did when they first made their discoveries, and similar exercises of spatial imagination play a role in the thinking of mechanical engineers, architects, designers of new furniture or tools, dress-makers, and many others who work on spatial structures, including inventing new, useful, types.

I suspect other intelligent animals can do something similar to a limited extent, but can't talk about it or reflect or their discoveries using an internal language, as humans can. I suspect this reasoning ability evolved before the development of language-based reasoning because it seems to exist in some non-human intelligent species without human languages, and some aspects of it are evident in pre-verbal human children (as illustrated in the video of child with pencil, mentioned above).

When I encountered the claim that mathematical knowledge fell into Hume's second category ("relations of ideas", i.e. definitional truths), thereby refuting Kant, I knew from my own experience of finding mathematical proofs e.g. proofs in geometry, that this argument against Kant was fallacious.

My 1962 DPhil thesis (now online) defended a slightly modified version of Kant's claim that many important mathematical discoveries are non-empirical, non-contingent, and non-analytic (i.e. not just logical consequences of axioms and definitions), but did not explain how brains or machines could make such discoveries.

There were several different sorts of argument, but a key part was to generalise the notion taken for granted by many logicians and philosophers since the work of Frege and Russell that sentences in which there are predicates and relations can be construed as applications of functions to arguments, a notion familiar from mathematics.

On this view the sentence "London is a city" applies the predicate "is a city" to the object London, and because that function produces the value TRUE, the statement made by the sentence is true.

Likewise if Jack and Jill are two individuals the sentence "Jack is shorter than Jill" is analysed as applying the two-argument function "... is shorter than
..." which could also be written
    Shorter(x,y)
to the individuals Jack and Jill. If the function produces the value TRUE then what is said is true. If the function produces the value FALSE, then what is said is false, but
    not(Shorter(x,y))
would be true.

In the case of mathematical functions, used in statements like
    six is greater than three
or in standard notation
    6 > 3
nothing that happens to be the case in the physical world can affect whether it is true or false. But in general we do need to look beyond the functions, and the arguments to which they are applied, to discover the value of a function. To check whether there are more blocks than balls on a table you need to know what blocks are, what balls are and how to compare numbers of objects. You don't need that capability in order to decide whether six is greater than three.

So the functions that are used as predicates and relation words in non-mathematical utterances typically have an implicit additional argument, namely the state of the universe (or a relevant portion of the universe).

Nevertheless there are many cases where we are able to tell that what is asserted in a proposition is incapable of being made false by the universe, or incapable of being made true.

Some of those are the cases that Hume described as merely expressing relations of ideas, or which we can regard as derivable from definitions using only logic.

But Kant's point was that in other cases what is said is incapable of having a different truth value (i.e. it is necessarily true or necessarily false) but not because of definitions and and their purely logical consequences.

An example which is centrally relevant to our ability to use the natural numbers is that we can use the relationship of two collections being in a one to one correspondence, e.g.

[apple banana elephant mouse] == [africa asia europe australia]

and

[africa asia europe australia] == [water salt wood smoke]

to infer that there is also a one-one correspondence if the items in one of the sets are reordered, and the correspondence will necessarily be preserved if any item in one of the sets is replaced by another not already in that set.

Moreover, every child learning these number concepts has to come to understand that the relation of one to one correspondence, is both transitive and symmetric, in order to understand the natural numbers. Moreover, neither property is merely an empirical property of the relation. (It is not merely an empirical generalisation.)

Research by Piaget suggests that such understanding does not come until year five or six in most young humans. So it is not innate, even if Kant is correct in saying that knowledge becomes non-empirical as a learner's understanding develops.

Neither is it just an empirical generalisation. Why not? See this draft interview (Sept 2019): https://www.cs.bham.ac.uk/~axs/kij-lars-aaron.pdf

NOTE (Added 16 Feb 2020)

> I am not a Hume scholar, but knowing how intelligent Hume was, I think it is possible that he did not restrict "relations of ideas" to *definitional* relations, but instead included the kinds of relations between ideas (of angle, length, radius, straightness, planarity, circularity, etc.) discovered by ancient mathematicians, some of which were assembled in a structured presentation in Euclid's *Elements*. Such discoveries could well be labelled *discoveries of (initially unobvious) relations between ideas*. In that case, Immanuel Kant was mistaken in his interpretation of Hume, which is just as well because that interpretation/misinterpretation provoked him to give a more detailed analysis of cases, and to raise important questions about mathematical cognition (that Hume may have ignored) to which we do not yet know the answers.

After being introduced to AI around 1969, by Max Clowes (Sloman/Clowes/1984), I learnt to program, and hoped to show how to build a baby robot that could grow up to be a mathematician making discoveries satisfying Kant's specifications, i.e. discoveries like those of Archimedes, Euclid, Zeno, etc., and many other deep discoveries made long before the development of modern logic and formal proof procedures.

Those mathematical abilities are a superset of, but depend on, the kinds of spatial intelligence in pre-verbal human toddlers, and other intelligent animals, e.g. squirrels, elephants, crows, apes, and perhaps octopuses[#] -- whose abilities are not yet replicated in AI/Robotics systems nor explained by current theories in neuroscience or psychology.
[#]
https://www.bbc.co.uk/iplayer/episode/m0007snt/natural-world-20192020-5-the-octopus-in-my-house

Insofar as such mathematical discoveries involve necessity or impossibility they *cannot* be substantiated by mechanisms that collect statistical information and derive probabilities.

This version of Kant's theory rules out natural and artificial neural nets and related forms of deep learning.

E.g. they cannot learn that something is impossible, such as a largest prime number, or a finite volume bounded by three plane surfaces. I have a large, and steadily growing, collection of examples to be explained by any adequate theory of mathematical consciousness.

I'll give more examples later.

Many more examples can be found here, and in documents referenced herein:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/impossible.html

Alan Turing's comments in his PhD thesis on the difference between mathematical intuition and mathematical ingenuity seem to me to echo Kant's insights, and I suspect (though the evidence is flimsy) that his 1952 paper on chemistry-based morphogenesis (nowadays his *most* cited paper) was at least partly motivated by a search for a new model of computation, combining continuous and discrete components. The most likely location for such a mechanism is sub-neural chemistry, for reasons related to Schrodinger's analysis in What is life? (1944) of the role of chemistry in reproduction. A few neuroscientists are exploring related ideas (e.g. Seth Grant in Edinburgh).

I'll present examples of spatial/mathematical reasoning illustrating Kant's claims. E.g. what sorts of brain mechanisms enable a child to understand that it's *impossible* to separate linked rings made of impermeable material? Why are you sure that no planar triangle can have one side whose length exceeds the combined lengths of the other two sides?) Current neurally inspired AI mechanisms cannot discover, or even represent, necessity or impossibility, or understand paragraphs like this. Logic-based mechanisms don't explain what was going on in mathematical brains before the development of logic in the last few centuries, or squirrel brains, or human toddler brains, e.g. this one: http://www.cs.bham.ac.uk/research/projects/cogaff/movies/ijcai-17/small-pencil-vid.webm
(Skip the introduction.)

The implications for the current wave of enthusiasm for deep learning are potentially devastating -- but invisible to people who have never studied Kant, or philosophy of mathematics. Which is not to deny that deep learning can be very useful, if used properly.

[xx] A disorganised collection of additional examples can be found here, with links to many more:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/impossible.html (also pdf)

This talk is part of the Artificial Intelligence and Natural Computation seminars series.

REFERENCES AND LINKS
(To be pruned later.)

H.G. Barrow and J.M. Tenenbaum, 1981, Interpreting Line Drawings as Three-Dimensional Surfaces, in *Artificial Intelligence,* 17, pp. 75--116, http://web.mit.edu/cocosci/Papers/Barrow-Tenenbaum81.pdf

Jordana Cepelewicz, 2016 How Does a Mathematician's Brain Differ from That of a Mere Mortal? *Scientific American Online* April 12, 2016
https://www.scientificamerican.com/article/how-does-a-mathematician-s-brain-differ-from-that-of-a-mere-mortal/

Jackie Chappell and Aaron Sloman (2007a). Natural and artificial meta-configured altricial information-processing systems. (2007a) *International Journal of Unconventional Computing*, 3(3), 211-239. http://www.cs.bham.ac.uk/research/projects/cogaff/07.html#717

Jackie Chappell and Aaron Sloman, (2007b) Two ways of understanding causation: Humean and Kantian,
Contributions to WONAC: International Workshop on Natural and Artificial Cognition Pembroke College, Oxford, June 25-26, 2007, http://www.cs.bham.ac.uk/research/projects/cogaff/talks/wonac

M.B. Clowes, 1973, Man the creative machine: A perspective from Artificial Intelligence research, in *The Limits of Human Nature*, Ed. J. Benthall, Allen Lane, London.

Chris Christensen (2013) Review of Biographies of Alan Turing, *Cryptologia*, 37:4, 356-367, https://doi.org/10.1080/01611194.2013.827532

Kenneth Craik, 1943, *The Nature of Explanation*, Cambridge University Press, London, New York
Craik drew attention to previously unnoticed problems about biological information processing in intelligent animals. For a draft incomplete discussion of his contribution, see http://www.cs.bham.ac.uk/research/projects/cogaff/misc/kenneth-craik.html

David Deutsch, 1997 *The Fabric of Reality,*
Allen Lane and Penguin Books

David Deutsch, 2011 *The Beginning of Infinity: Explanations That Transform the World,*
Allen Lane and Penguin Books, London. Euclid and John Casey (2007) *The First Six Books of the Elements of Euclid,* Project Gutenberg, Salt Lake City, Third Edition, Revised and enlarged. Dublin: Hodges, Figgis, \& Co., Grafton-St. London: Longmans, Green, \& Co. 1885, http://www.gutenberg.org/ebooks/21076

H. Gelernter, 1964, Realization of a geometry-theorem proving machine, reprinted in *Computers and Thought,* Eds. Edward A. Feigenbaum and Julian Feldman, McGraw-Hill, New York, pp. 134-152,
http://dl.acm.org/citation.cfm?id=216408.216418

Robert Geretschlager, 1995. Euclidean Constructions and the Geometry of Origami, *Mathematics Magazine,* 68, 5, pp. 357--371, Mathematical Association of America, http://www.jstor.org/stable/2690924

James J. Gibson, 1979 *The Ecological Approach to Visual Perception,* Houghton Mifflin, Boston, MA,

Ira Goldstein, 1973, Elementary Geometry Theorem Proving MIT AI Memo 280, April 1973
https://dspace.mit.edu/handle/1721.1/5798

Yacin Hamami and John Mumma, 2013, Prolegomena to a Cognitive Investigation of Euclidean Diagrammatic Reasoning, in *Journ Log Lang Inf* 22, pp 421-448
http://doi.org/10.1007/s10849-013-9182-8

Carl G. Hempel, Geometry and Empirical Science, 1945, *American Mathematical Monthly*, Vol 52, Reprinted in *Readings in Philosophical Analysis*, eds. H. Feigl and W. Sellars, New York: Appleton-Century-Crofts, 1949,
http://www.ditext.com/hempel/geo.html

David Hilbert, 1899, *The Foundations of Geometry,*, available at Project Gutenberg, Salt Lake City, http://www.gutenberg.org/ebooks/17384 2005, Translated 1902 by E.J. Townsend, from 1899 German edition,

Andrew Hodges, 1999. *Turing*. New York: Routledge.

T. Ida and J. Fleuriot, Eds., *Proc. 9th Int. Workshop on Automated Deduction in Geometry (ADG 2012)*, Edinburgh, September, 2012, University of Edinburgh, Informatics Research Report, http://dream.inf.ed.ac.uk/events/adg2012/uploads/proceedings/ADG2012-proceedings.pdf

Immanuel Kant's *Critique of Pure Reason* (1781)
has relevant ideas and questions, but he lacked our present understanding of information processing (which is still too limited)
http://archive.org/details/immanuelkantscri032379mbp

Imre Lakatos, *Proofs and Refutations,*
Cambridge University Press, 1976,

John McCarthy and Patrick J. Hayes, 1969, "Some philosophical problems from the standpoint of AI", *Machine Intelligence 4,* Eds. B. Meltzer and D. Michie, pp. 463--502, Edinburgh University Press,
http://www-formal.stanford.edu/jmc/mcchay69/mcchay69.html

Kenneth Manders (1998) The Euclidean Diagram, reprinted 2008 in *The Philosophy of Mathematical Practice*, OUP, pp.80--133 *Ed* Paolo Mancosu,
http://doi.org/10.1093/acprof:oso/9780199296453.003.0005

Kenneth Manders (2008) "Diagram-Based Geometric Practice", In Paolo Mancosu (ed.), *The Philosophy of Mathematical Practice.* OUP, pp.65--79
http://doi.org/10.1093/acprof:oso/9780199296453.003.0004

D. Mareschal and M. S. C. Thomas, "How computational models help explain the origins of reasoning," in *IEEE Computational Intelligence Magazine*, vol. 1, no. 3, pp. 32-40, Aug. 2006. doi: 10.1109/MCI.2006.1672986
http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=1672986&isnumber=35104

Noboru Matsuda and Kurt Vanlehn (2004), GRAMY: A Geometry Theorem Prover Capable of Construction, *Journal of Automated Reasoning* Vol 32 (3--33) Kluwer Academic Publishers. Netherlands.
http://www.public.asu.edu/~kvanlehn/Stringent/PDF/04JAR_NM_KVL.pdf

David Mumford, 2016, Grammar isn't merely part of language, *Online Blog,*
http://www.dam.brown.edu/people/mumford/blog/2016/grammar.html

Tuck Newport, *Brains and Computers: Amino Acids versus Transistors,*
2015, Kindle,
Discusses implications of href="#von-Neumann-brain">von Neumann 1958,
https://www.amazon.com/dp/B00OQFN6LA

Jean Piaget, (1952). *The Child's Conception of Number.* London: Routledge & Kegan Paul.

Piaget, 1981, 1983 Jean Piaget's last two (closely related) books written with collaborators are relevant, though I don't think he had good explanatory theories.
Possibility and Necessity
Vol 1. The role of possibility in cognitive development (1981)
Vol 2. The role of necessity in cognitive development (1983)
University of Minnesota Press, Tr. by Helga Feider from French in 1987

(Like Kant, Piaget had deep observations but lacked an understanding of information processing mechanisms, required for explanatory theories.)

Gualtiero Piccinini (2003), Alan Turing and the Mathematical Objection, *Minds and Machines* Feb, 2003, Kluwer Academic
http://dx.doi.org/10.1023/A:1021348629167

L. J. Rips, A. Bloomfield and J. Asmuth, 2008, From Numerical Concepts to Concepts of Number, *The Behavioral and Brain Sciences,* Vol 31, no 6, pp. 623--642,
https://doi.org/10.1017/S0140525X08005566

Carl J. Posy (Ed), 1992 *Kant's Philosophy of Mathematics-Modern Essays* Synthese Library Vol 219 Kluwer Academic Publishers

Erwin Schrödinger (1944) *What is life?* CUP, Cambridge,
I have an annotated version of part of this book here (also PDF):
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/schrodinger-life.html

Dana Scott, 2014, Geometry without points. (Video lecture, 23 June 2014,University of Edinburgh)
https://www.youtube.com/watch?v=sDGnE8eja5o

Frege on the Foundation of Geometry in Intuition *Journal for the History of Analytical Philosophy* Vol 3, No 6. pp 1-23,
https://jhaponline.org/jhap/issue/view/271

Siemann, J., & Petermann, F. (2018). Innate or Acquired? - Disentangling Number Sense and Early Number Competencies. Frontiers in psychology, 9, 571. doi:10.3389/fpsyg.2018.00571
https://www.ncbi.nlm.nih.gov/pubmed/29725316

Sloman, A. (1962). *Knowing and Understanding: Relations between meaning and truth, meaning and necessary truth, meaning and synthetic necessary truth* (DPhil Thesis), Oxford University. (Transcribed version online.)
http://www.cs.bham.ac.uk/research/projects/cogaff/62-80.html#1962

Aaron Sloman (1963/5) Functions and Rogators, In *Formal Systems and Recursive Functions: Proceedings of the Eighth Logic Colloquium Oxford, July 1963,* Eds. J. N. Crossley and M. A. E. Dummett, North-Holland, Amsterdam, 1965, pp. 156--175.
http://www.cs.bham.ac.uk/research/projects/cogaff/07.html#714

Aaron Sloman, 1965, "Necessary", "A Priori" and "Analytic", *Analysis*, Vol 26, No 1, pp. 12--16.
http://www.cs.bham.ac.uk/research/projects/cogaff/62-80.html#1965-02

A. Sloman, 1971, "Interactions between philosophy and AI: The role of intuition and non-logical reasoning in intelligence", in *Proc 2nd IJCAI,* pp. 209--226, London. William Kaufmann. Reprinted in *Artificial Intelligence,* vol 2, 3-4, pp 209-225, 1971.
http://www.cs.bham.ac.uk/research/cogaff/62-80.html#1971-02
A slightly expanded version was published as chapter 7 of Sloman 1978, available here.

A. Sloman, 1978 *The Computer Revolution in Philosophy,*
Harvester Press (and Humanities Press), Hassocks, Sussex.
Free, partly revised, edition online:
http://www.cs.bham.ac.uk/research/cogaff/62-80.html#crp

A. Sloman, (1978b). What About Their Internal Languages? Commentary on three articles by Premack, D., Woodruff, G., by Griffin, D.R., and by Savage-Rumbaugh, E.S., Rumbaugh, D.R., Boysen, S. in BBS Journal 1978, 1 (4). *Behavioral and Brain Sciences,* 1(4), 515.
http://www.cs.bham.ac.uk/research/projects/cogaff/62-80.html#1978-02

Aaron Sloman (1984--2018) Experiencing Computation: A Tribute to Max Clowes, in *New horizons in educational computing,* Ed. Masoud Yazdani, Ellis Horwood Series in AI, pp. 207--219. (Online version with expanded obituary and biography.)
http://www.cs.bham.ac.uk/research/projects/cogaff/sloman-clowestribute.html

Aaron Sloman (2012-...), The Meta-Morphogenesis (Self-Informing Universe) Project (begun 2012, with several progress reports, but still work in progress).
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-morphogenesis.html
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-morphogenesis.pdf

Aaron Sloman, 2015-18, Some (possibly) new considerations regarding impossible objects, (Their significance for mathematical cognition, current serious limitations of AI vision systems, and philosophy of mind, i.e. contents of consciousness), Online research presentation,
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/impossible.html

Aaron Sloman, 2013--2018, Jane Austen's concept of information (Not Claude Shannon's)
Online technical report, University of Birmingham,
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/austen-info.html
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/austen-info.pdf

Aaron Sloman, 2016, Natural Vision and Mathematics: Seeing Impossibilities, in *Proceedings of Second Workshop on: Bridging the Gap between Human and Automated Reasoning*, IJCAI 2016, pp.86--101, Eds. Ulrich Furbach and Claudia Schon, July, 9, New York,
http://ceur-ws.org/Vol-1651/
http://www.cs.bham.ac.uk/research/projects/cogaff/sloman-bridging-gap-2016.pdf

A. Sloman (with help from Jackie Chappell), 2017-8, The Meta-Configured Genome (unpublished)
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-configured-genome.html

A. Sloman, 2018a, A Super-Turing (Multi) Membrane Machine for Geometers Part 1
(Also for toddlers, and other intelligent animals)
PART 1: Philosophical and biological background
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/super-turing-phil.html

A. Sloman, 2018b A Super-Turing (Multi) Membrane Machine for Geometers Part 2
(Also for toddlers, and other intelligent animals)
PART 2: Towards a specification for mechanisms
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/super-turing-geom.html

Aaron Sloman, 2018c,
Biologically Evolved Forms of Compositionality
Structural relations and constraints vs Statistical correlations and probabilities
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/compositionality.html (also PDF).
Expanded version of paper accepted for *First Symposium on Compositional Structures* (SYCO 1)
Sept 2018 School of Computer Science, University of Birmingham, UK
http://events.cs.bham.ac.uk/syco/1/

Draft interview (Sept 2019) about how I got into AI and the consequences:
https://www.cs.bham.ac.uk/~axs/kij-lars-aaron.pdf
Notes for an interview to be published in an AI journal.

Wikipedia contributors, Tarski's axioms for geometry *Wikipedia, The Free Encyclopedia,*
[Accessed 6-November-2018]
https://en.wikipedia.org/wiki/Tarski%27s_axioms

Trettenbrein, Patrick C., 2016, The Demise of the Synapse As the Locus of Memory: A Looming Paradigm Shift?, *Frontiers in Systems Neuroscience,* Vol 88,
http://doi.org/10.3389/fnsys.2016.00088

A. M. Turing, (1950) Computing machinery and intelligence,
Mind, 59, pp. 433--460, 1950,
(reprinted in many collections, e.g. E.A. Feigenbaum and J. Feldman (eds)
Computers and Thought McGraw-Hill, New York, 1963, 11--35),
WARNING: some of the online and published copies of this paper have errors,
including claiming that computers will have 109 rather than 10^9 bits
of memory. Anyone who blindly copies that error cannot be trusted as a commentator.

A. M. Turing, (1952), 'The Chemical Basis Of Morphogenesis', in
Phil. Trans. R. Soc. London B 237, 237, pp. 37--72.
(Also reprinted(with commentaries) in S. B. Cooper and J. van Leeuwen, EDs (2013)).

A useful summary of Turing's 1952 paper for non-mathematicians is:
Philip Ball, 2015, Forging patterns and making waves from biology to geology: a commentary on Turing (1952) 'The chemical basis of morphogenesis', *Royal Society Philosophical Transactions B,*
http://dx.doi.org/10.1098/rstb.2014.0218

John von Neumann, 1958 *The Computer and the Brain* (Silliman Memorial Lectures), Yale University Press. 3rd Edition, with Foreword by Ray Kurzweill. Originally published 1958.

Wikipedia contributors, 2018, Mathematics of paper folding *Wikipedia, The Free Encyclopedia,*
https://en.wikipedia.org/w/index.php?title=Mathematics_of_paper_folding&oldid=862366869

Alastair Wilson, 2017, Metaphysical Causation, *Nous*
https://doi.org/10.1111/nous.12190

Stephen Wolfram (2007), Mathematics, Mathematica and Certainty *Wolfram Blog* December 8, 2007
http://blog.wolfram.com/2007/12/08/mathematics-mathematica-and-certainty/

Maintained by Aaron Sloman
School of Computer Science
The University of Birmingham