This is part of the
MORCOM Plenary Workshop
(MORphological COMputing of Cognition and Intelligence)
Which is part of IS4SI:
2021 Summit of the International Society for the Study of Information
https://summit-2021.is4si.org/

THIS WEB SITE IS STILL UNDER CONSTRUCTION AND LIKELY TO CHANGE
These were originally notes for the first hour of
a three hour meeting using Zoom, with two breaks,
Starting Wed 15th Sept 2021 (13:00 UCT 14:00 BST)
These notes will be used for a repeat recording.

Because I could not share my screen on 15th Sept, the chairperson had to show copies of my
prepared items. I have now re-enabled screen sharing and this document will be used for a new
recording, in the near future, after which this comment will be removed.
These are (incomplete) notes for a repeat recording of the 1st hour of the meeting on 15th Sept.

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Speakers on 15th Sept 2021:
Aaron Sloman
Why Don’t Hatching Alligator Eggs Ever Produce Chicks? (Details below)
http://www.cs.bham.ac.uk/~axs/
School of Computer Science, University of Birmingham

Michael Levin, Tufts University, will reply.
Morphogenesis as a model for computation and basal cognition
https://as.tufts.edu/biology/people/faculty/michael-levin
Talk Abstract

Chairperson:
Gordana Dodig-Crnkovic (Chalmers University)
Professor of Computer Science at Mälardalen University
and Professor of Interaction Design at Chalmers University of Technology
Her notes for this meeting are available here.
The two speakers had an hour each, followed by a discussion, approximately 1 hour.

This was a MORCOM workshop plenary session on 15th Sept
Full IS4SI conference schedule:
https://summit-2021.is4si.org/schedule

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DRAFT NOTES FOR SLOMAN PRESENTATION
Why Don’t Hatching Alligator Eggs Ever Produce Chicks?

Animal spatial intelligence and limits of current AI and neuroscience. With implications for philosophy of mathematics, philosophy of mind, theories of evolution, theories of morphogenesis during reproduction, metaphysical/scientific theories about mechanisms involved in reproduction including reproduction of spatial and other forms of intelligence.

Most of MORCOM was on the day after the plenary session: Thursday 16th Sept:
https://summit-2021.is4si.org/schedule/morcom-schedule

A copy of the conference schedule with associated documents is available here:
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/IS4SI_MORCOM_IWNC

This talk was originally presented on: Wednesday September 15 2021
Because A.Sloman had a screen-sharing problem on the day, his session will be re-recorded, and a link provided on this web site.

This document is
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/sloman-morcom.html
(Work in progress and liable to change -- please do not quote.)
Last updated: 24 Sep 2021

Plenary Session Timetable on 15th Sept 2021

Sloman Presentation: 13:00-14:00 UTC (14:00-15:00 BST) 15th Sept 2021
(Detailed notes for the presentation are below.)

Levin Response: 14:00-15:00 UTC (15:00-16:00 BST)
Some of his relevant work is here:
https://ase.tufts.edu/biology/labs/levin/

The two presentations were followed by a discussion
15:00-16:00 UTC (16:00-17:00 BST)
Moderator/Chair: Gordana Dodig-Crnkovic

{} Brief overview of this presentation (Still work in progress)
There is a vast amount of research on various aspects of morphogenesis and self-organising abilities of living organisms, such as slime-molds and more recently xenobots (presented in Michael Levin’s talk).

As far as I can tell, NO current theory answers the specific questions discussed below concerning how chemical mechanisms involved in reproduction (discussed later) contribute to forms of spatial intelligence in many animals, including competences required for ancient mathematical discoveries, discussed below.
Levin’s work on xenobots has very promising novel features, though it seems unlikely that the current ideas can account for all the multi-level phenomena described below. He may have other relevant work of which I am ignorant.

My talk summarises and combines many strands of research over many years, since completion of my Oxford DPhil thesis (Sloman(1962)), in which some of the key problems, originally posed by Immanuel Kant around (1781) were discussed -- a discussion continued in several later publications, including Sloman(1978). Kant’s ideas about mathematical discovery, triggered in response to David Hume, are an important part of the background to this talk, as explained below.

Related talks
A precursor to this talk was presented, using Zoom, at the University of Sussex, on 16th Feb, 2021, as described here: http://www.cs.bham.ac.uk/research/projects/cogaff/misc/sloman-chemneuro-sussex.html
(Including a link to the recording.)
A revised version was presented and discussed (via zoom) at an event hosted on 2nd June 2021, by Tubingen University. A recording of the talk, followed by a recording of the discussion: https://www.youtube.com/channel/UCaG1Q8TEuLN5OJZXaTL28PQ
https://www.cs.bham.ac.uk/research/projects/cogaff/movies/tub/as-discuss.mp4
A shorter, less detailed, slightly chaotic presentation was given at the PhiloWeb 2021 Workshop on 22nd June 2021, as part of the 13th ACM Web Science Conference 2021, at which my talk starts shortly after 25:00 and ends at 55:00 in this recording of the complete day-long event: https://www.cs.bham.ac.uk/research/projects/cogaff/misc/sloman-PhiloWeb.html

What follows is a substantially revised version of the previously presented material. The ideas are still under continuous development and this document is likely to change after the presentation on 15th September 2021.

NOTES FOR PRESENTATION ON 15 SEPT 2021
Updated: 25 Sep 2021

{} Some background information: developments inside eggs
I’ll have a lot to say about how developments inside eggs lead to theories of cognition that are very different from current popular theories based on properties of neural networks. Neural nets are not available to control most of the processes in which organisms develop inside eggs.

The following image crudely indicates stages of development in a chicken egg (which are far more complex than shape changing slime mold, or current xenobot, behaviours):
Here’s a video showing some of what goes on inside a chicken egg during the hatching process: https://youtu.be/PedajVADLGw

Courtesy of: Cobb Ltd

Some of the stages

DAY 1: Appearance of embryonic tissue.
DAY 2: Tissue development very visible. Appearance of blood vessels.
DAY 4: Eye pigmented.
DAY 5: Appearance of elbows and knees.
DAY 8: Feather tracts seen. Upper and lower beak equal in length.
DAY 9: Embryo starts to look bird-like. Mouth opening occurs.
DAY 10: Egg tooth prominent. Toe nails visible.
DAY 11: Cob serrated. Tail feathers apparent.
DAY 12: Toes fully formed. First few visible feathers.
DAY 14: Embryo turns head towards large end of egg.
DAY 15: Gut is drawn into abdominal cavity.
DAY 16: Feathers cover complete body. Albumen nearly gone.
DAY 17: Amniotic fluid decreases. Head is between legs.
DAY 18: Growth of embryo nearly complete. Yolk sac remains outside of embryo. Head is under...
right wing.
DAY 19: Yolk sac draws into body cavity. Amniotic fluid gone. Embryo occupies most of space within egg (not in the air cell).
DAY 20: Yolk sac drawn completely into body. Embryo becomes a chick (breathing air with its lungs). Internal and external pipping occurs.

My thanks to: [https://www.backyardchickens.com/](https://www.backyardchickens.com/)

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**Details not shown in such images and videos**

The description of the hatching process summarised above says much about the division of different parts of the embryo into new sub-types, but very little about the huge amount of very intricate interconnection going on inside the developing animal, leading to production of bones, nerve fibres, heart, lungs, veins, arteries, and capillary networks each of which has complex and precise relationships to other structures growing in the embryo.

For example, both blood vessels and nerve fibres are distributed over all parts of the developing organism. Blood vessels need to be developed to carry blood to and from nearly all parts of the body, using arteries and veins transmitting blood in opposite directions with capillary networks fanning out from arteries and back into veins. Groups of veins and groups of arteries can perform different sorts of functions depending on where they are located and which major organs they interact with, e.g. picking up or delivering nutrients, oxygen, hormones, waste products, and thermal energy.

All the physiological substructures are functionally related to others, in some cases only to immediate neighbours (e.g. muscles exerting forces on body-parts to which they are attached) and in other cases long distance transfer of nutrients or information, using chemicals or electrical signalling.

The increasingly differentiated components of an embryo perform a wide, and changing, variety of functions, both during development of the embryo and later, while interacting with the environment after hatching.

These functions include both transfer of types of physical matter such as nutrients, waste products and chemical signals (e.g. hormones) and transfer of information, some of it based on chemical signals some not.

During development of an embryo inside an egg, there is a steady increase in diversity, complexity, and types of function of the components of the foetus. There are also many increases in diversity and complexity of types of communication and coordination between components.

These processes are richer/more complex than the processes in shape-changing organisms such as slime mold, mentioned above.

Michael Levin’s talk will demonstrate processes that could be more relevant to the formation of a complete new organism with many new intricately interrelated functioning parts, as opposed to a mere shape-changing organism.
He mentions the example of metamorphosis: one physical form (e.g. caterpillar) transforming itself through chemical rearrangements into another form (e.g. butterfly). This process must include mechanisms of the sort discussed here, insofar as the emerging butterfly has not only a completely new physical form, but also competences related to flying and mating that were not relevant to its previous form.

But so far I don’t think the mechanisms discussed by Levin, or any other proposals I have encountered, are capable of explaining all the developmental phenomena described below, including production of human mathematical abilities based on ancient forms of spatial reasoning, described by Immanuel Kant, in opposition to David Hume’s ideas, summarised later.

I’ll try to explain why after explaining the title of this talk!

{}  
Why this title: Why Don’t Hatching Alligator Eggs Ever Produce Chicks?
The above title was an off-the-cuff response to an invitation several months before the conference. A less obscure alternative could be:

_How do chemical processes in eggs produce both complex physiological structures and various types of spatial intelligence in new hatchlings?_

I shall try to show how that question is linked, in surprising ways, to some deep metaphysical problems, concerning what is and is not possible in this universe, and why ..... which in turn is related to the nature of mathematics, especially ancient forms of mathematics involving geometry and topology.

I’ll give reasons for thinking that the mechanisms involved in understanding impossibility and necessity, including examples of spatial necessity and impossibility discovered by ancient mathematicians, long before Euclid, cannot be based on probabilistic neural nets.

They are more likely to depend crucially on chemical information processing machinery, although I cannot yet specify details. That remains a major research problem. I’ll present some clues below.

Immanuel Kant’s philosophy of mathematics indicates that he had a deep understanding of the key points I am making. I’ll defend some of his ideas that are now unfashionable for bad reasons.

{}  
_Ancient mathematics_

In humans, mechanisms of spatial cognition enabled ancient mathematicians, centuries before Euclid, to make discoveries regarding possibility, impossibility and necessity in spatial structures and processes, without making use of modern mathematical formalisms in which theorems are derivable using only logical inferences: instead they used spatial forms of reasoning, e.g. reasoning based on real or imagined spatial structures rather than logical/algebraic formalisms.
Some intelligent non-human animals (e.g. squirrels, some nest-building birds, octopuses) also seem to have such (species specific) abilities. They can make use of spatial possibility, impossibility, and necessity in selecting and performing actions.

However, they don’t have the additional ability that some humans have, namely to communicate their knowledge to others, and may not have brain mechanisms required for reflecting on what is known, or expressing such knowledge in a linguistic format.

Young children also have, and make use of, much knowledge of spatial possibilities and impossibilities that they cannot articulate verbally, but can use in choosing, rejecting or reflecting on possible actions. Additional examples of “toddler theorems” can be found in Sloman (2013). Piaget’s last two books (1981, 1983), published posthumously, on Possibility and Necessity explored such abilities in children of various ages. However, he was unable to propose explanatory mechanisms.

Many philosophers of mathematics seem to believe that everything that could be done using ancient spatial reasoning mechanisms can be replicated in modern discrete, logic-based, reasoning systems. It’s possible that the ancient forms can be modelled in the new forms, but without being replicated (e.g. using physical mechanisms with similar speeds and energy costs).

During the discussion of this point after my presentation in a previous conference, Pat Hayes commented that a great deal of geometry and topology has been expressed in logic-based formal systems using only discrete symbols and discrete operations on symbols. The most obvious and well known example is the great mathematician David Hilbert’s "axiomatization" of Euclidean geometry Hilbert (1899).

I am not claiming that discrete, logic-based formalisms cannot be used to replicate (or model) ancient geometric discoveries. I claim only that there are alternative mechanisms using spatial reasoning rather than symbolic manipulations, that were used by ancient mathematicians, engineers, architects and also ordinary folk, long before humans discovered the space of formal systems developed in the 19th and 20th Centuries, including Hilbert’s logic-based formalisation of the subset of geometry captured by Euclid’s informal axiomatisation. I suspect, but cannot (yet) demonstrate, that those older mathematicians were using chemical mechanisms related to the chemical mechanisms used to control complex assembly processes in eggs.

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Mathematical contents of everyday spatial intelligence.  
Mathematical (but not meta-mathematical) insights involving topology and geometry are involved in surprising ways in the competences of very young children and many highly intelligent non-human animals, although they are not able to think about their intelligence: they lack required forms of meta-cognition. Examples are presented below.

Moreover, currently fashionable neuroscience and neurally inspired AI are totally incompetent at explaining such abilities, because they focus mainly on discovering and using statistical information and derived probabilities, ignoring discoveries of necessity and impossibility, which cannot be derived from statistical evidence.
For example, how can a child who has not studied topology at school understand the transitivity of "contains", or the impossibility of linking or unlinking, without damaging, solid rings? I'll return to such questions below.

Here's an example involving two stone rings forming a (short) chain on an old Indian temple:

**Fig: Stone Rings**

![Stone Rings](http://en.wikipedia.org/wiki/Group_of_temple%5Fes_at_Talakad%5F_India_%5FKarnataka)

Each ring appears to be made from a single piece of solid, rigid, stone.

*Could the two rings have been cut out of two separate rigid blocks of stone and then assembled as shown?*

**How do you know the answer to this?**

NOTE: A comment on this by Luc Beaudoin is **below**.

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**Bad answers, and better answers**

Currently fashionable neural net theories explain only types of knowledge involving probabilities that are derived from previously collected statistical evidence. But explanations of probabilities cannot be explanations of necessity or impossibility.

Therefore such neural mechanisms cannot explain forms of spatial reasoning used by ancient mathematicians, for instance in several hundred different proofs of Pythagoras’ theorem discovered by mathematicians centuries before Pythagoras was born -- all demonstrating that in a right angled triangle the area of the square on the hypoteneuse is necessarily, i.e. it MUST BE, equal to the sum of the squares on the two smaller sides. It is impossible for the third side to be longer or shorter than that sum, assuming the triangle is on a flat surface, e.g. not on the surface of a sphere.
or a teapot.

An example proof is available on this Wikipedia page
https://en.wikipedia.org/wiki/Pythagorean_theorem. Click on the box on the right of that page for a demonstration.

For similar reasons, neural nets cannot explain the spatial abilities of intelligent species such as squirrels, crows, elephants, pre-verbal humans, that involve recognition of necessity or impossibility, about which I'll say more below in commenting on differences between the philosophies of David Hume and Immanuel Kant.

Less obviously, trainable neural networks cannot explain the spatial intelligence of newly hatched creatures, like the foraging competences of young avocets in this video clip from a BBC Springwatch programme[*] in June 2021.
https://www.cs.bham.ac.uk/research/projects/cogaff/movies/avocets/avocet-hatchlings.mp4

Processes inside eggs cannot train neural nets to interact with the waterside environment since that environment is inaccessible in the egg. And, after hatching, the chicks clearly do not need to train their neural networks before they walk to water and start foraging. How are those competences implemented in their brains and what mechanisms put them there?

The full Springwatch episode, showing the avocet hatchlings, is on Youtube at:
https://www.youtube.com/watch?v=FV6ZHe0CiHw
The section on “Avocet Island” starts at about 12min 23sec. The above 35 second extract, showing competences of newly hatched avocets, starts at about 12mins 30secs.

{} Steps toward better answers
The competences of newly hatched birds must have been produced by in-egg pre-hatching processes using unknown chemistry-based mechanisms. What else could be available inside eggs?

What additional mechanisms could help? Below, I'll discuss conjectured chemistry-based reasoning mechanisms whose details are still unknown.

There are many aspects of common-sense spatial reasoning that are concerned with impossibility and necessity. E.g. containment in space is necessarily transitive. So the search for explanatory mechanisms presented below is relevant to far more than mathematical competences, as Immanuel Kant pointed out in his Critique of Pure Reason (1781). I'll give a tiny subset of examples -- far more are available online, e.g. in http://www.cs.bham.ac.uk/research/projects/cogaff/misc/impossible.html and documents referenced therein.

Neural net mechanisms are assumed by many researchers to provide the explanations of mathematical and other forms of cognition.

But those researchers fail to realise that neural nets cannot represent, let alone prove, spatial impossibility or necessity, since they merely collect statistics and derive probabilities. Necessity is not equivalent to an extremely high probability.
Statistics-based neural nets therefore cannot match or explain important aspects of human and non-human spatial intelligence based on detection or creation of examples of impossibility, necessity, or possibility.

Modern logic-based reasoning can establish necessity or impossibility, but most of the relevant reasoning processes were discovered by humans only in the last few centuries. There is no evidence that they were used by the ancient geometers, or other intelligent animals, centuries before Euclid. Hilbert.

I'll suggest below that currently unknown chemical mechanisms may suffice. Moreover, I'll give reasons for thinking that related

{} Deformed triangle example
I'll demonstrate a simple example of necessity/impossibility involving a deformable triangle. If a straight line passes through a vertex of a triangle and through the interior of the triangle and the opposite side of the triangle and the vertex is moved along that line towards or away from the opposite side, what happens to the size of the angle at the vertex?

What happens to the size of the angle A, on triangle ABC, if A is moved away from the opposite side (BC), along a fixed line that passes through the opposite side, between vertices B and C, until A reaches location A'. If the locations of vertices B and C do not change while A is moved further away, does the size of angle A increase, decrease or remain unchanged?
How do you know? What sort of brain makes such reasoning possible?
Is it simply an empirical generalisation that may be refuted next week?
A more complex discussion, related to an ancient geometric theorem of Apollonius is here: 
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/deform-triangle.html
A graphical demonstration will illustrate spatial necessities/impossibilities associated with possible deformations of a triangle constrained by a straight line.

There are many more examples in this document: 
http://www.cs.bham.ac.uk/research/projects/cogaff/misc/impossible.html
(PDF version also available.)

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As Immanuel Kant implied in his (1781), necessity and impossibility are not very high and very low probabilities. "Necessarily true" and "Impossible" are not points on probability scales. I'll say more about Kant, and his opposition to Hume, below.

{}  
Moreover, recently developed, logic-based, formal reasoning mechanisms account for only a subset of mathematical discoveries. They cannot explain spatial reasoning abilities of ancient humans, e.g. ancient pyramid builders and ancient mathematicians, or even pre-verbal children, like this crawler with a pencil

Exploring topology/holes

GIF VIDEO (TEMPORARILY DISABLED)  
child+pencil+hole

The above child (age about 17.5 months) seems to be exploring topology. She spontaneously crawled towards the sheet of card while holding a pencil, picked up the card, pushed the pencil through the hole, pulled the pencil out, moved the pencil up and over the edge of the card while rotating the card toward the pencil then pushed the pencil through the hole from the opposite side, then removed the pencil, reverted to the original side and finally pushed the pencil in then pulled it out again.

The above 'gif video' may not work for you in this context. The episode can also be viewed in this video, which includes a commentary and some slow motion: small-pencil-vid.webm

{}  
The abilities of pre-verbal toddlers, and many other intelligent animals, seem to indicate an intuitive (proto-mathematical) grasp of possibilities, impossibilities and necessary connections, though the children do not notice and cannot describe their own competences. This is also true of most adults. It takes a reflective mind like that of Immanuel Kant to notice surprising features of some human powers of reasoning and discovery. His ideas are summarised and contrasted with David Hume's, below,

The well known physicist, Roger Penrose, mentioned below, has made related claims about important kinds of mathematical discovery, but without specifying explanatory brain mechanisms, though he suggests that the mechanisms cannot be Turing-equivalent forms of computation, and must use quantum physics. For very different reasons, another physicist Erwin Schrödinger (1944) pointed out that reliable biological reproduction depends on quantum physics.
Below I’ll suggest that some unnoticed facts about reproductive processes in eggs also depend on quantum mechanisms, that Alan Turing may have been thinking about before he died.

**A familiar (adult) "domestic" example**

I suspect many people have encountered the impossibility of pushing a wide armchair through a doorway that’s too narrow, and the creation of a new collection of possibilities by turning the armchair on its side, which (for some chairs) makes it possible to push the chair through the doorway while rotating it about a vertical axis.

Children learn to manipulate spatial necessity and impossibility by playing with appropriate toys, including construction kits -- of varying richness. E.g. contrast Tinker Toys with Meccano.

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**How did ancient (and not so ancient) mathematical brains achieve their discoveries?**

I suggest that hitherto unnoticed chemistry-based mechanisms, required for biological assembly, obviously required in eggs, also underpin some complex, species-specific, forms of intelligence. An example is the intelligence of newly-hatched animals, e.g. the avocet hatchlings shown in the BBC Springwatch programme on 1st June, mentioned above.

I suspect that a similar conjecture formed the unstated motivation for Alan Turing’s work on chemical morphogenesis (1952) in the last few years before his death in 1954. If so, his 1952 paper on chemistry-based pattern formation was just a digression from his unstated deeper research problem. There is a brief hint about this in the sentence about brain chemistry in his well known 1950 paper (‘Computing machinery and intelligence’).

What chemistry-based mechanisms can enable mathematical reasoning about spatial structures and processes? How do they do it? I suspect nobody knows at present.

Is current theoretical physics sufficiently rich to explain all the phenomena I’ll be talking about today, or all the geometric phenomena presented by Michael Levin in his talk? There’s some overlap in our interests though so far I don’t think he has attempted to describe or explain ancient forms of mathematical cognition. Perhaps his ideas will turn out relevant to that in surprising ways.

**More detailed questions:**

How do the competences and explanatory mechanisms differ in production of hatchlings of different species, such as the young of different bird species, and also baby alligators or turtles, which have very different physical forms and very different capabilities.

Are there any key features that the different species-specific mechanisms have in common?

In particular, for different species, what chemical processes completely enclosed in eggs can determine both construction of complex physical forms (including extremely intricate multi-material, internal physiology) and also the competences used in complex, species-specific, physical behaviours, often produced very soon after hatching --- unmatched by current robots? (Unless there have been recent developments I’ve not heard about.)
Production, within an egg, of physical components, e.g. bones, tendons, muscles, glands, nerve fibres, skin, hair, scales, or feathers, and other structures, including intricate networks of blood vessels, nerve-fibres and other physiological structures, is clearly chemistry-based, and far more complex than chemistry based behaviours of shape changing organisms, such as slime molds, in ways that are not immediately obvious, which I’ll try to explain in the presentation.

Moreover, the combination of complexity, compactness, energy-efficiency, and speed of production of processes in an egg are also unmatched by human designed assembly-lines.

The recent work on "xenobots" (to be reported in Michael Levin’s talk) may be an important (early?) step toward adequate explanatory mechanisms. But, as far as I know, they have not yet been shown to be capable of producing hatchlings with the combination of spatial intelligence and complex and intricate, multi-material, physiological structures created in eggs.

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**Gene-expression is not a uniform process**

EARLY STAGES of gene expression that are common across species that use DNA for reproduction are well understood and at first involve only one cell, then a few cells,... [Provide link or reference]

But as a developing foetus becomes more complex, with millions of cells of many kinds, the gene-expression requirements and mechanisms become more complex (because more complex changes become possible) and also become more specific to the type of organism. E.g. later stages of gene expression in a developing foetus will be very different in a flea, an earthworm, a chicken, an alligator, a dandelion, a giant redwood tree, and so on.

Moreover, even in a particular individual, as it becomes more complex with more different body parts, the processes of gene expression in different parts will be different, though different parts may need to be coordinated -- e.g. development of bones, muscles, tendons, nerve-fibres, arteries, veins, glands, skin, mouth, waste exits, etc., and the coordination requirements will be different for adjacent parts, e.g. parts of a limb, or adjacent vertebrae, and for non-adjacent parts, e.g. linkage of blood vessels between heart, lungs, brains, digestive system, and other body parts.

Moreover, the cross-body coordination requirements will be different for different species. For example, some vertebrate eggs produce organisms with no legs (e.g. snakes) some with two legs and two wings (e.g birds), some with four legs (e.g. horses, crocodiles), some with two legs and two arms, and so on.

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**Increasing kinds of long-distance coordination**

It is unlikely that all that coordination can be achieved simply by chemical processes reacting to the contents of their immediate neighbourhoods -- in complex organisms increasingly complex forms of cross-organism coordination are required as the embryo develops, and they will need to change as sizes, weights, strengths, distances, etc. change.

The later processes of gene expression will produce highly complex species-specific physical configurations and forms of coordination between body parts, while still in the egg, and will have to continue extending them after hatching as the new hatchling’s sensor and body parts are used in interaction with different sorts of objects in different environments -- e.g. while sheltered under the hen, while walking across dry land, while moving into water and starting to paddle or swim, and
while pecking or diving for food, then manipulating it in beak or mouth, then swallowing, etc. How are all these extraordinarily complex assembly processes controlled? Notice that because the physiological structures, body sizes, environments, behaviours and food needs differ across species, the mechanisms of co-ordination and control will also differ, and will need to be constructed differently for different species.

### Developing requirements for centralised control

I cannot see how all those interactions can simply be emergent cumulative results of many local interactions between body parts. Brains, eyes, mouth or beak and leg muscles need to be coordinated in appropriate ways in the presence of static food, catchable prey or a threatening predator. Different forms of cross-body coordination are needed during in-egg development processes, subsequent growth after hatching or birth, and interactions with resources and dangers in the environment.

I’ll suggest that the later stages of development in eggs are controlled by a succession of increasingly complex virtual machines with hitherto unknown, non-space-occupying mechanisms, whose construction needs to be boot-strapped via multi-layered assembly processes that are far more complex than anything achieved in human designed assembly plants.

In particular, the later more sophisticated virtual machines required to control assembly are not directly specified in the genome, but have to be boot-strapped by earlier stages that produce new mechanisms required for the later stages.

### The meta-configured genome (MCG) hypothesis

All the above comments illustrate features of what Jackie Chappell and I call "A meta-configured genome", about which I’ll say more below. (There are also several freely available online documents and videos about these ideas that can be found using internet searches.)

A feature of the MCG hypothesis is that different assembly mechanisms are required for assembly at different stages of development of a foetus. So the assembly mechanisms need to produce new assembly mechanisms while they produce new physical/chemical components in the foetus.

Another key feature is that the specifications for later stages of development, especially development of information processing systems for dealing with perception, motivation, action-selection, planning, control of actions, and accumulating various kinds of knowledge about the environment and its contents may be left partly unspecified by the genome, so that missing specifications can be derived or constructed by making use of information that is acquired during various stages of development.

In particular, some of the motive-generation machinery, instead of being specified in the genome may have to be developed in relation to each individual’s environment, using "architecture-based-motivation" mechanisms e.g. those proposed in Sloman(2009) (or its later revisions).
These mechanisms enable young individuals to discover that it is good to be motivated by certain perceived opportunities, because of the benefits gained by acting on those opportunities. So the genome itself does not specify all the top level goals that are useful for individual members of the species. Instead they can depend on the environment.

To illustrate this: the kinds of motivation that were useful for children of ancient cave dwelling humans are very different from the kinds of motivation that are useful for young humans developing in 2021, when the opportunities and benefits are totally different.

Eggs vs factories

The processes in eggs, however, use far less matter and energy in their operation than processes in human designed factories producing complex machinery.

Developing explanatory theories will need new forms of multi-disciplinary collaboration, and are likely to have profound implications for theories of brain function, replacing current theories that cannot explain ancient mathematical discoveries.

I suspect it will turn out that current physical theory needs to be extended to explain how the processes sketched below can occur in eggs.

The mechanisms must be primarily chemistry-based, since neurons are created relatively late by chemical mechanisms in eggs. Powerful chemical assembly mechanisms play important roles in bodies and brains throughout development -- performing tasks that neural nets cannot perform, including underpinning mathematical insight.

We need an entirely new form of brain science giving far more credit to chemical processes whose computational powers exceed those of both digital computers and neural nets.

Is that why Alan Turing was exploring chemistry-based morphogenesis shortly before he died? Turing(1952)

Some key ideas
(to be reorganised)

(There will not be time for all of this at the workshop.)

There are aspects of spatial intelligence in humans and other animals, that are ignored in most current theories in neuroscience, psychology, philosophy of mind philosophy of mathematics, and also in all the AI models that I know of.

These aspects include abilities to detect and make use of varieties of spatial possibility, necessity and impossibility.

Currently fashionable "neural net" - based models, collect statistical evidence and derive probabilities. They are therefore constitutionally incapable of discovering or representing impossibility or necessity, which are not extremes of probability, but, as kant pointed out, are key features of ancient mathematical discoveries.
Logic-based models of geometric reasoning fail to model human spatial intelligence in different ways -- but most were not intended as models of cognition.

Around 1781, Immanuel Kant, reacting to David Hume’s account of types of knowledge had some deep insights -- and at at least partly understood the difficulty of finding complete explanations. There’s more on Kant below.

We still cannot explain all the processes and mechanisms involved in spatial reasoning in humans and other animals, which provided the infrastructure for ancient mathematical discoveries in geometry and topology, centuries before Pythagoras and Euclid. However, I’ll present some ideas about the problem as steps toward a possible (future) answer. There is evidence that Alan Turing was working on related ideas shortly before he died. His 1952 paper on chemistry-based morphogenesis was about development of 2D surface patterns. I think that was a fairly shallow side-track in a much deeper investigation that he failed to complete before he died (in 1954).

I am no Turing, but I can make some observations and vague (meta-level) suggestions about assembly processes in eggs that may help future researchers to explain production of hatchlings with significant spatial competences required for moving around, finding, and eating food.

The ideas are also potentially relevant to animals produced in wombs, where the division between the developing embryo and active individual is not so sharp.

Some of the proposed mechanisms are inspired by features of complex, multi-layered mechanisms such as the internet, and the extent to which its functioning depends on profound use of increasingly complex and sophisticated forms of virtual non-space-occupying machinery, implemented in frequently changing physical machinery. However, the conjectured control processes eggs will need to be far more complex and subtle than anything created so far by human engineers.

A similar comment can be made about how mechanisms that make possible processes of cognition in humans and other species are related to brain mechanisms. If the key ideas of this paper are correct, sub-neural chemical processes in brains will turn out to be far more important in explaining aspects of human spatial intelligence, including ancient mathematical intelligence, than statistics-based neural mechanisms.

Some intelligent non-human animals with powerful spatial reasoning abilities may turn out to have more in common with ancient mathematicians than anyone has noticed.

What happens inside a bird’s egg?
Some of the things that go on inside eggs must involve powerful mechanisms for assembling very complex/intricate structures, like the body of a newly hatched chick, including many microscopic and sub-microscopic subsystems such as the circulatory system (including arteries, veins, capillary networks, and interfaces to many organs that interact with and through that system) along with many other subsystems providing different facilities, such as the structural roles of bones, muscles, ligaments, etc.

I’ll also present evidence that those chemical assembly mechanisms not only produce extremely complex physical/physiological structures (and functions) but (in some species) also provide information-processing capabilities required for performing complex actions in the environment shortly after hatching, actions using competences that many theorists seem to assume must be the
result of learning what does and does not work in the environment, e.g. by training neural networks.

For many researchers, the most surprising claim in this presentation will be that capabilities that are often thought to be the result of learning by interacting with the environment (e.g. to train neural networks) can instead be the products of chemical assembly processes in eggs -- processes that as far as I can tell nobody understands at present. I shall relate this to Immanuel Kant’s theories about the nature of mathematical intelligence.\textsuperscript{(1781)}

Initially the mechanisms controlling gene expression are chemical interactions within a single cell, but as the mechanisms being added or extended grow more complex and more spatially extended (across multiple cells of different types), the machinery for controlling the development must also grow more complex.

That seems to require the processes controlling development within the egg to be repeatedly extended by producing new layers of control mechanism. These new mechanisms obviously cannot take up extra space in the egg (which would require a space-occupancy explosion).

So perhaps evolution discovered, long before humans did, the importance of virtual machinery controlling large physical machines, as illustrated by many internet mechanisms, such as email systems, banking systems, online reservation systems, which share a great deal of physical infrastructure, but perform different functions using that infrastructure. Moreover, the virtual machines that run on physical internet hardware are constantly being extended and diversified.

Sometimes this can be done without significant physical changes to internet mechanisms. But in other cases the speed and capacity of internet mechanisms need to be extended to support new virtual machinery -- e.g. high speed games in virtual environments played by humans on different continents.

**Egg-based future-proofing machinery!**

An important feature of processes inside an egg is that in many species the in-egg processes constructing a new embryo produce individuals with complex forms of spatial intelligence that are required for interacting with the immediate environment after hatching, without any special training or learning, as illustrated by the avocets shown in the video above.

So the mechanism-building machinery in the egg must be able to create not only new physical structures in the embryo but also powerful new information processing mechanisms for use after hatching, including genetically provided information about quite detailed aspects of the environment, and how they can be made use of by performing appropriate actions.

Could the post-hatching mechanisms for controlling behaviours in external spaces share some of the pre-hatching mechanisms for controlling relatively large scale assembly and coordination processes within the egg, since both involve abilities to make use of information about spatial relationships?

I suggest that those abilities are not built into a single monolithic spatial understanding machine inside the egg, but assembled from a variety of spatial reasoning mechanisms that evolved at different times, and are used at different stages of development -- including constructing later layers of virtual machinery.
There are some well-known and striking examples of organisms capable of complex shape changes triggered by features of the environment, such as the behaviour of slime mold shown in many online tutorial videos, such as [https://www.youtube.com/watch?v=40f7_93Ni5A](https://www.youtube.com/watch?v=40f7_93Ni5A) with (over-optimistic?) caption “What self-driving cars can learn from brainless slime mold”!

However, there are deep differences between behaviours of slime molds (as I understand them) and the chemical construction processes that go on inside eggs, where the global shape (of the egg) does not change much, apart from growth in size and deviation from sphericity, but the intricate internal details constantly become more complex and more diverse in a growing collection of highly parallel processes.

**The control and coordination of all those intricately interrelated structures and processes is not a uniform control process.**

As the foetus grows more complex, the tasks become more varied, more complex, and straddle more scales, e.g. over time sizes of structures that need to be related become larger, and distances between connections increase.

So in parallel with construction of new parts of the organism, there must be construction of new mechanisms controlling assembly. But there is no space inside the egg for extra controlling machinery (like the robotic arms alongside production-lines in human-built factories).

So it appears that biological evolution found ways to construct new virtual machines that can control assembly processes without occupying additional space!

Could such biological virtual machines share some features with the increasingly complex and varied virtual machines added to the internet that operate to perform new tasks without changing the physical structure of the internet -- e.g. setting up a zoom conference connecting pre-existing computers using pre-existing internet connections and user peripherals?

Of course, in the embryo there are also physical processes constructing new parts or changing shapes and sizes of existing parts as the organism develops, so that requires additional complexity beyond what the internet needs.

I don’t yet know enough to decide whether the mechanisms described in Levin’s work suffice to explain all of that, or whether additional ideas are required, perhaps even revisions of fundamental physics to explain the possibility of such high speed, low energy, richly controlled, multi-layer virtual-machine assembly controllers.

**Note on "minimal" conditions for life (Tibor Ganti)**

A topic that is not discussed in this document, but is closely related to the discussion of the roles of varieties of information in varieties of types of life, is the question whether there is a well defined "minimal" combination of features required for life. One of the best-known and most interesting attempts to answer this question is the work of Tibor Ganti(2003), although the requirements he specified are not met by viruses, for example. Whether viruses should be classed as a form of life is to some extent an uninteresting question of definition though how they are similar to and different from other entities that are regarded as definitely alive or definitely not alive is of some interest -- as discussed in this BBC tutorial: [https://www.bbc.co.uk/bitesize/articles/zkcvhcw](https://www.bbc.co.uk/bitesize/articles/zkcvhcw)
This talk does not attempt to draw a boundary. My topic is more a matter of characterising differences between entities that do and do not make use of information of different kinds for different purposes in different contexts, e.g. for growth, reproduction, protection, and other biological functions. But there are different views on what information is, and on that point I am definitely not using “information” in the sense defined by Shannon.

**Not Shannon Information!**
Control requires use of information to determine what needs to be done at each stage.

This paper makes repeated essential use of the notion of "information". However, it is important to stress that I am not talking about Shannon information, a concept that goes back to about 1944, and refers to information bearing capacities in physical or virtual structures, not to their semantic content.

There is a very much older semantic concept of information used repeatedly by Jane Austen in her novels, a century before Shannon, as discussed in Sloman *(2013--2018)* (and other papers referenced therein).

Unfortunately, Shannon’s choice of the label "information" (rather than something like "information-capacity") has misled and confused many thinkers in many disciplines, though Shannon himself was not at all confused.

**Is whole-organism development what Alan Turing was really interested in when working on morphogenesis?**

Turing’s much-cited 1952 paper on the chemical basis of morphogenesis mentioned below was about development of 2D surface patterns. I suspect that paper was merely an interim progress report on a much deeper problem concerned more with development of 3D biological structures (as discussed above), and chemical mechanisms for thought processes, though it seems that he did not wish to provide information on work in progress in the 1952 paper. He died two years later without publishing anything more on morphogenesis, as far as I know. I suspect Turing’s analysis of chemically controlled embryo development would have gone much further and deeper than Schrödinger’s in *(1944)*.

I think Turing had unwittingly rediscovered Kant’s problem and begun to develop an answer. Perhaps we’ll never know how far he had got by the time he died. This talk presents a few small steps in the direction of providing answers.

There’s a tangled web of processes going on inside a developing embryo, with increasing levels of complexity as new layers of structure and control develop.

The earliest stages of morphogenesis are well studied and understood, including propagating local interactions among molecules within a cell. But we need something much richer and more powerful to explain later stages of development as both the number but also the variety of types of cell explode.

I think it is useful to apply what we have learnt about the importance, and diversity, of types of virtual machinery, designed and deployed since mid 20th Century. The earliest examples were merely software simulations of one (possibly not yet built) physical computer in another computer.
But the complexity, variety, and types of application of virtual machinery have continually expanded since then, and especially during the last two decades, driven by the increasing power and variety of applications using the internet -- most recently virtual conferences using tools like zoom, but also banking systems, multi-national company information systems, airline reservation systems, many kinds of sales and marketing systems, government information systems, flight control systems, and many more.

I suggest that biological evolution discovered and made sophisticated use of virtual machinery long before we did, notably in processes of reproduction of and control of increasingly complex organisms, and in some cases collections of organisms.

The evolving concept of "virtual machinery"
Far more sophisticated forms of virtual machinery have been developed for use in providing services across the internet that "float persistently" above the constantly changing particular physical mechanisms at work, but without occupying additional space.

E.g. think of all the (constantly changing) physical processes supporting an ongoing Zoom-based workshop by routing messages across multiple virtual pathways using possibly changing physical infrastructure.

Did evolution "discover" the possibility, and powers of such machinery long before human engineers did? What are the implications for current theories of basic physics?

And biology, neuroscience, philosophy, ...

As far as I know, there is no human-designed structure, e.g. no major chemical plant, whose products have diversity and complexity of internal structure and function comparable to a developing embryo in a chicken egg during later stages of hatching. Perhaps not even the largest chemical plants match the internal structural and functional complexity of an egg during the later stages of embryo development?

And there is no combination of known technologies that could assemble a machine as complex as a newly hatched chick in a space (including assembly machinery) as small as a chicken’s egg, or even the space of a larger egg, such as a crocodile egg! Moreover nothing with comparable complexity of structure and function could be assembled in three weeks, using current human engineering mechanisms, and consuming so little energy in the process!

How many years would it take for us to build machines that replicate the functionality of a single chicken egg? Can it be done using current technology? Could a comparable much larger self-organising egg-like machine be built using current scientific knowledge and technology? I suspect not.

Another question raised by all this, is whether there are aspects of current physical theory, especially the theory of very low-energy physics, that need to be extended in order to explain the details of reproduction of chickens and other animals.

How are the assembly processes controlled?
A major challenge is to explain how production of all that internal diversity is controlled so that things grow into the "right" sets of relationships, where parts of some objects e.g. lungs, heart, are functionally closely related to many other parts, e.g. all the detailed portions of veins, arteries and
capillary networks.

In addition to creation of physical structures, the processes of development in the egg result in provision of sophisticated spatial competences (e.g. perception, reasoning, action selection, control of action) in the chick, or other animal, available shortly after hatching, as illustrated by the avocet video above.

Currently popular theories in psychology, neuroscience and AI that I have encountered, attempting to explain complex abilities to interact with objects in the environment, assume that everything has to be learnt by acting in the environment and collecting information about what does and does not work, etc. This is the basis of neural net models of learning. (Not all psychologists share this view, e.g. Piaget, referenced below.)

Most theorists I have encountered who claim to explain spatial cognition and spatial skills seem to me to underestimate deeply the roles of innate (but self-extending) mechanisms using powerful ancient forms of virtual machinery -- not yet understood, though I think Alan Turing was working on relevant ideas shortly before he died, and Kant seems to have understood some of the requirements for explanations that most 21st Century theorists ignore.

Post-hatching displays of intelligence in avocet chicks

The video-clip above illustrates the commentator’s remark about behaviours soon after hatching: “They head down to the water straight away and they start to forage”. This happens without any process of learning to walk, to find water, and to walk or paddle about in the water detecting prey items which they catch and swallow.

What are the implications of the fact that such complex competences can be provided for the new organism while it is being grown inside an egg?

How is that possible? There are questions about how such mechanisms evolved, how they develop inside a hatching egg, and how they actually work: how are the competences represented in the genome? How is that information used in producing a new physical implementation? How does that produce complex useful interaction with the environment shortly after hatching, or shortly after birth in some other species, e.g. foals that run with the herd to escape a predator shortly after birth.

What does all this imply about how much explanatory progress has been made in current AI and neuroscience?

More Examples of spatial - non-discrete reasoning:

As is obvious to young children, it is impossible to separate two linked rings made of solid impenetrable material simply by moving them around in space, without breaking open either of the rings.

Curves on the surface of a torus

(Skip during conference presentation)

In a planar or spherical 2D surface S, if C is a (non-self crossing) closed curve, then C divides the surface S into two non-overlapping portions, S1 and S2, and every continuous line L in the surface that joins a point in S1 and a point in S2 *must* also cross the curve C.
If S is a toroidal surface, e.g. the surface of a ring, then the above is true for some closed curves in S but not all. (Think of closed curves drawn on the surface of the inflated inner tube of a car wheel.)

Now consider the figure below, containing five curves, B1, B2 both blue, R1 red, and Y1, Y2 yellow. These are all closed curves: they have no free ends.

You can probably tell that the two yellow closed curves Y1 and Y2 are mutually continuously transformable: each can be smoothly moved in the surface of the torus to occupy the exact location of the other. How do you convince yourself that it is possible? Do you have to physically create a succession of intermediate curves, or is it enough to imagine them? Do you have to imagine all of the intermediate locations? Is the equivalence obvious at a more abstract level? What brain mechanisms could discover such obviousness?

**A challenge**
Suppose there are two non-self-crossing continuous closed curves C1 and C2 on the surface of a torus, we can say they are in the same "equivalence class" if C1 can be continuously deformed into C2. Is it possible for C1 to be continuously deformable into C2, but not vice versa? How could you know such asymmetry is impossible? A new question arises out of this: **How many equivalence classes of continuous closed curves are there on the surface of a torus?** What sort of brain makes it possible for you to work out the answer simply by thinking about the problem, without having to be trained on thousands, or millions, ... or an infinite supply ... of different sets of curved lines on curved surfaces? Does any current robot have an artificial brain with such powers? Could you design such a robot?

**Cem Tezer’s online presentations**
There are still modern mathematicians who present proofs that are based on spatial reasoning, supplemented with verbal commentaries, such as the wonderful demonstrations by mathematician Cem Tezer in these two video presentations on properties of triangles and related structures:
The great modern physicist, Roger Penrose, has also been exploring and attempting to explain such ancient human reasoning capabilities for many years, as illustrated in these lectures, and many others available online:

https://www.youtube.com/watch?v=eKjZ2W7eXlo
https://www.youtube.com/watch?v=th3YMEamzmw
https://www.interaliamag.org/audiovisual/roger-penrose-how-drawing-is-used-for-maths-and-science/

I have not encountered a specification for an explanatory brain mechanism in his work.

I suspect (and I think Turing also thought) that those ancient, and not so ancient, spatial reasoning processes cannot all be replicated/modelled in logic based systems, including digital computers, because they use chemistry-based information processing mechanisms combining discrete and continuous processes (a conclusion I did not reach until writing a paper for a Turing centenary volume). Roger Penrose implicitly makes similar claims, but as far as I can tell has not yet presented any explanatory mechanism.

In effect, Immanuel Kant, as explained below, made related claims about some forms of mathematical reasoning in 1781, but my claims in this presentation do not depend on whether purely symbolic, logic-based, reasoning cannot replicate the results of the ancient forms of mathematical reasoning.

I claim only that there were ancient (and not so ancient) mathematicians whose geometric reasoning was not based solely on use of logic and definitions, and I conjecture, partly because of the spatial reasoning capabilities of many newly hatched animals, such as the avocets mentioned above, that many mathematical brains use chemistry-based forms of representation and reasoning.

Their discoveries could not have been based on the use of neural nets, since those merely collect statistics and derive probabilities. They cannot establish necessity or impossibility. They cannot even represent those concepts.

(My comments don’t apply to hybrid systems in which neural nets are combined with other mechanisms, e.g. logical theorem provers.)

**Kant vs Hume**

Immanuel Kant’s characterisation of ancient mathematical knowledge, in his Critique of Pure Reason (1781) drew attention to three features of such cognition, using three distinctions that are ignored in current psychology, neuroscience and neural-net based AI: the non-empirical/empirical, analytic/synthetic, and necessary/contingent distinctions.
Very crudely, David Hume, depicted above, on the left, claimed that there are only two kinds of knowledge:

1. **empirical** knowledge that comes through sensory mechanisms, possibly aided by measuring devices of various kinds; which he called knowledge of "matters of fact and real existence";
2. what he called "relations of ideas", which we can think of as things that are true by definition, such as "All bachelors are unmarried", and (if I've understood Hume rightly) all mathematical knowledge, for example ancient knowledge of arithmetic and geometry, which Hume’s words seemed to suggest was no more informative than the bachelor example.

"True by definition" applies to all truths that can be proved using only logic and definitions.

An example is "No bachelor uncle is an only child", which can easily be proved from the definitions of "bachelor", "uncle" and "only child", using only logical reasoning.

Hume famously claimed that if someone claims to know something that is neither of type 1 (empirical) nor of type 2 (mere relations between ideas, or definitional truths) we should "Commit it then to the flames: for it can contain nothing but sophistry and illusion", which would have included much theological writing. and much philosophical writing by metaphysicians.

[I apologise to Hume and Hume scholars: this presentation over-simplifies Hume’s position in order to contrast it with Kant’s claims, below.]

**Immanuel Kant’s response (1781)**

In response to Hume, Immanuel Kant, depicted above, on the right, claimed, in his *Critique of Pure Reason*, that there are some important kinds of knowledge that don’t fit into either of Hume’s two categories ("Hume’s fork"), for they are not mere matters of definition, nor derivable from definitions by using logic.
Kant pointed out that instead of Hume's single distinction between two categories of knowledge we need to take account of three different distinctions:

- the analytic/synthetic distinction,
- the empirical/non-empirical (empirical/apriori) distinction, and
- the necessary/contingent distinction.

(For a more detailed explanation of the three distinctions see Sloman 1965).

Using Kant's distinctions, we can locate ancient mathematical discoveries in relation to three different contrasts:

- The mathematical truths such as Pythagoras' theorem and others proved by Euclid are not analytic but synthetic, i.e. they are not provable simply using definitions and logic -- for example spatial reasoning is required. and

- They also are not derived from sensory experiences by generalising from examples, in such a way that if the experiences had been different the knowledge acquired would have been different, like the knowledge that eating berries with a certain appearance can make you feel ill. You cannot discover that fact simply by reasoning about berries, whereas mathematical discoveries e.g. about numbers or triangles can be discovered by reasoning, so they are apriori not empirical. (Note that "apriori" does not imply "innate". It merely rules out knowledge being derived from observations of the world that could have yielded different results, as is true of most of our knowledge about the world.)

- Moreover, the world could not have been different in ways that would have made our mathematical discoveries false: they are necessarily true, not contingently true, like propositions that are true but could have been false, such as the proposition that we are in the midst of a Covid-19 pandemic. That could have been false, perhaps if certain human behaviours had been different.

Another example: I am now in Birmingham in England. In principle I could now have been somewhere else at this time, e.g. in Berlin, in Germany. So that is a contingent truth.

If something is a necessary truth, then there are no possible circumstances in which it could be false.

There are also necessary falsehoods. E.g. $3 + 5 = 9$ is false and could not have been true in any circumstances (without changing the meaning of what is being said. So it is necessarily false and its negation is necessarily true.

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In short: Kant replaced Hume's single division of types of knowledge into two categories, with a much richer analysis making use of three different divisions, producing six categories. Not all combinations are possible, however. E.g. something cannot be both apriori and necessarily false.
People who have been deprived of the traditional type of mathematical education normally fail to discover these distinctions themselves, and may not understand them if they read about them in Kant, or commentators.

Both logic-based and neural-net-based AI systems are incapable of replicating the ancient aspects natural intelligence that allow discoveries based on spatial reasoning. Moreover, the ancient mathematical capabilities described by Kant are usually ignored in currently fashionable theories and models of mathematical cognition in psychology, neuroscience, philosophy and AI, including theories that focus on social aspects of mathematics.

Theories at odds with Kant’s insights include both ‘formal’, logic-based, characterisations of mathematics, used in modern automated theorem provers, that reason by manipulating discrete symbolic structures, and also neural theories that attempt to explain or model mathematical discovery processes in terms of neural networks that collect statistical evidence that is used to derive probabilities. Necessity and impossibility are not extremes on probability scales.

As Kant realised, ancient knowledge of geometry, (including mathematical discoveries made centuries before Euclid), is neither simply composed of results of empirical generalisation from experience of special cases (i.e. empirical knowledge of probabilities), nor mere logical consequences of definitions. I.e. they are non-empirical (a priori), and synthetic (not derivable from definitions using only logic, and the truths they identify are non-contingent, despite some claimed counter-examples.

It is less obvious that he was also right about arithmetical knowledge, insofar as ancient number knowledge was derived from properties of the one-to-one correspondence relation, e.g. the fact that the one-to-one correspondence relation is necessarily transitive and symmetric -- which young humans seem to be unable to grasp before the age of five or six, as shown by Piaget in (1952).

[Compare Piaget’s later work on children’s understanding of possibility and necessity (1981,1983). I don’t think he ever succeeded in formulating explanatory theories.]

This refutes theories about innateness of knowledge of cardinality, unlike knowledge of numerosity, which lacks the precision of cardinality. The numerosity of a visible part of the environment can be thought of as roughly the product of the average density (e.g. of leaves visible on a tree, or light-points visible in a portion of the night sky) and the proportion of the visual field occupied. That product gives an approximate but unreliable estimate of the actual number of distinct items visible. It can be applied to scenes where there are too many objects to count, such as leaves visible on a tree or stars/planets visible in a region of the sky at night. The numeracy estimate does not make use of one-to-one correspondence.
My talk will raise questions about mechanisms available for explaining spatial intelligence in humans and other animals based on hitherto unexplained facts about spatial competences of newly hatched animals, such as chicks, ducklings, turtles and crocodiles, whose abilities cannot be explained by neural networks trained after hatching. They must be explained by chemical mechanisms inside their eggs, available before hatching. I’ll raise questions about the nature of those mechanisms linking many different sorts of parts of a developing organism and how they can be accommodated inside a fully occupied egg.

Construction kits built during development (epigenesis)

A key feature of the Turing-inspired Meta-Morphogenesis project, introduced here: http://www.cs.bham.ac.uk/research/projects/cogaff/misc/meta-morphogenesis.html is the use of increasingly complex evolved construction kits: mechanisms for creating objects of a particular (more or less general) type -- including construction-kits for building new construction-kits Sloman 2014(+many updates).

Some new construction kits are products of evolution of a species and are initially shared only between a few members of the species, alongside other species-specific and cross-species construction kits (shared between species), such as those used in mechanisms of reproduction and growth in related species.

Evolution also seems to have discovered the benefits of "meta-construction-kits": mechanisms that allow members of a species to build new construction kits during their own development. One of the most striking examples is language development in humans: which often includes individual variation.

Examples include mechanisms for learning that are initially generic mechanisms shared across individuals, and developed by individuals on the basis of their own previously encountered learning experiences, which may be different in different environments for members of the same species.

Human language learning is a striking example: things learnt at earlier stages make new things learnable that might not be learnable by an individual transferred from a different environment, part way through learning a different language.

This contrast between genetically specified and individually built capabilities for learning and development was labelled a difference between "pre-configured" and "meta-configured" competences in [Chappell Sloman 2007], summarised below in the Meta-configured Genome section and in in Figure EVO-DEVO.

The meta-configured competences are partly specified in the genome but those partial, abstract, specifications are instantiated in combination with information abstracted from individual experiences in various domains, of increasing abstraction and increasing complexity. We originally offered this conjecture as a more general alternative to Waddington’s notion of "an epigenetic landscape”, which was thought of as providing a fixed collection of developmental options for all members of a species.

Mathematical development and language development in humans both seem to be special cases of growth of such meta-configured competences.
Meta-Configured genomes

Multiple routes from genome to behaviours

A particular collection of construction kits specified in a genome can give rise to very different individuals in different contexts if the genome interacts with the environment in increasingly complex ways during development, allowing enormously varied developmental trajectories. Precocial species use only the downward routes on the left, producing only "preconfigured" competences. Competences of members of "altricial" species, using staggered development, may be far more varied within a species. Results of using earlier competences interact with the genome, producing "meta-configured" competences shown on the right. This is a modified version of a figure in Chappell & Sloman (2007).

Construction kits used for assembly of new organisms that start as a seed or an egg enable many different processes in which components are assembled in parallel, using abilities of the different sub-processes to constrain one another. Nobody knows the full variety of ways in which parallel construction processes can exercise mutual control in developing organisms. One implication of Figure EVO-DEVO is that there are not always simple correlations between genes and organism features.

The main idea could be summarised approximately as follows:

Instead of the genome determining how the organism reacts to its environment, the environment can cumulatively determine how the genome expresses itself: with different sorts of influence at different stages of development. This should not be confused with theories that attempt to measure percentages of genetic vs environmental influence in individual development. Numerical measures in this context are much shallower than specifications of structures and their interactions. Compare: expressing the percentage of one composer’s
influence on another (e.g. Haydn’s influence on Beethoven) would give little understanding of what the later composer had learnt from his or her predecessor. Often emphasising measurement over precise description can obfuscate science instead of deepening it. Likewise emphasising correlations can get in the way of understanding mechanisms.

Explaining the many ways in which a genome can orchestrate parallel processes of growth, development, formation of connections, etc. is a huge challenge. A framework allowing abstract specifications in a genome to interact with details of the environment in instantiating complex designs is illustrated schematically in Fig. 3.

An example might be the proposal in [Popper 1976] that newly evolved desires of individual organisms (e.g. desires to reach fruit in taller trees) could indirectly and gradually, across generations, influence selection of physical characteristics (e.g. longer necks, abilities to jump higher) that improve success-rates of actions triggered by those desires.

Various kinds of creativity, including mathematical creativity, might result from such transitions. This generalises Waddington’s "epigenetic landscape" metaphor [Waddington 1957], by allowing individual members of a species to partially construct and repeatedly modify their own epigenetic landscapes instead of merely following paths in a landscape that is common to the species. Mechanisms that increase developmental variability may also make new developmental defects possible (e.g. autism?)

2.4 The variety of biological construction kits

As products of physical construction kits become more complex, with more ways of contributing to needs of organisms, and directly or indirectly to reproductive fitness, they require increasingly sophisticated control mechanisms. New sorts of control often use new types of information. Processing that information may require new mechanisms. That may require new construction kits for building new types of information processing mechanism.

The simplest organisms use only a few types of (mainly chemical) sensor, providing information about internal states and the immediate external physical environment. They have very few behavioural options. They acquire, use and replace fragments of information, using the same forms of information throughout their life, to control deployment of a fixed repertoire of capabilities.

More complex organisms acquire information about enduring spatial locations in extended terrain, including static and changing routes between static and changing resources and dangers. They need to construct and use far more complex (internal or external) information stores about their environment, and, in some cases, "meta-semantic" information about information processing, in themselves and in others, e.g. conspecifics, predators and prey.

What forms can all the intermediate types of information take? Many controlled systems have states that can be represented by a fixed set of physical measures, often referred to as "variables", representing states of sensors, output signals, and internal states of various sorts. Such systems have many engineering applications, so many researchers are tempted to postulate them in biological information processing. Are they adequate?

\[ http://www.cs.bham.ac.uk/research/projects/cogaff/misc/autism.html \]
Relationships between static and changing state-components in such systems are often represented mathematically by equations, including differential equations, and constraints (e.g. inequalities) specifying restricted, possibly time-varying, ranges of values for the variables, or magnitude relations between the variables. A system with N variables (including derivatives) has a state of a fixed dimension, N. The only way to record new information in such systems is through static or changing values for numeric variables -- changing "state vectors", and possibly alterations in the equations.

There are many well understood special cases, such as simple forms of homeostatic control using negative feedback. Neural net based controllers often use large numbers of variables clustered into strongly interacting sub-groups, groups of groups, etc. Are these structures and mechanisms adequate for all biological information processing -- including human perception and reasoning?

For many structures and processes, a set of numerical values and rates of change linked by equations (including differential equations) expressing their changing relationships is an adequate form of representation, but not for all.

That's why chemists use *structural* formulae, e.g. diagrams showing different sorts of bond between atoms, and collections of diagrams showing how bonds change in chemical reactions.

Linguists, programmers, computer scientists, architects, structural engineers, map-makers, map-users, mathematicians studying geometry and topology, composers, and many others, work in domains where structural diagrams, logical expressions, grammars, programming languages, plan formalisms, and other *non-numerical* notations express information about structures and processes that is not usefully expressed in terms of collections of numbers and equations linking numbers.

**David Deutsch Constructor Theory**

I believe there are connections between the ideas here and the ideas being developed by David Deutsch and his colleague Carla Maietto with the label "Constructor theory", but I have not yet looked closely enough to understand the relationship apart from the fact that we both emphasise the importance of the scientific study of *possibilities* rather than the study of *laws*. (See Sloman (1978, Chapter 2))

**Note:**

Luc Beaudoin pointed out that the question posed in Fig: Stone Rings depends on whether blocks of stone can be continuously deformed, e.g. when heated. When I originally posed the question I had implicitly, unwittingly, excluded that possibility, forgetting that the materials of which blocks of stone are made can be deformed if pressure and temperature are sufficiently high, as happens below the surface of this planet! (Though it's unclear that the label "block of stone" would still be applicable in such a context.) Adding "rigid" to the question makes the exclusion explicit. The history of mathematics includes many mistakes arising from such oversights, as illustrated superbly in Lakatos (1976). The possibility of such oversights implies that Kant's claims about the nature of mathematics should not be interpreted as implying that mathematical reasoning is infallible, as pointed out in Sloman (1962).
CLOSELY RELATED RESEARCH
To be extended

Minimal Intelligence Lab (MINT Lab)
Studying Plant Intelligence (and other forms)

http://www.um.es/web/minimal-intelligence-lab/
http://www.um.es/web/minimal-intelligence-lab/contenido/manifesto
Expanded Manifesto (PDF):
http://www.um.es/documents/2103613/2107123/MANIFESTO_PLANT+NEUROBIOLOGY+AND+ITS+PHILOSOPHY.pdf
The Minimal Intelligence Team:
http://www.um.es/web/minimal-intelligence-lab/contenido/the-team
“Frontiers” page on Minimal intelligence across Eukaryota

http://www.cs.bham.ac.uk/research/projects/cogaff/07.html#717

Abstract:
The full variety of powerful information-processing mechanisms ‘discovered’ by evolution has not yet been re-discovered by scientists and engineers. By attending closely to the diversity of biological phenomena, we may gain new insights into

(a) how evolution happens, including how it extends the mechanisms of evolution by evolving new construction-kits, meta-construction-kits, meta-meta-construction-kits, etc.
(b) what sorts of mechanisms, forms of representation, types of learning and development, and types of architectures have evolved,
(c) how to explain ill-understood aspects of human and animal intelligence, and perhaps, if our technology is sufficiently extendable:
(d) new useful mechanisms for artificial systems.

We analyse trade-offs common to both biological evolution and engineering design, and propose a kind of architecture that grows itself, using, among other things, genetically determined meta-competences that deploy powerful symbolic mechanisms to achieve various kinds of discontinuous learning, often through play and exploration, including development of an ‘exosomatic’ ontology, referring to things in the environment - in contrast with learning systems that discover only sensorimotor contingencies or adaptive mechanisms that make only minor modifications within a fixed architecture. We sometimes refer to this collection of ideas as “The Meta-Configured genome”. Paul Davies (2012) seems to have developed related ideas, though without the theory of construction-kits.

REFERENCES
(A small sample, for now.)
https://doi.org/10.1007/s11569-020-00377-1

Paul C. W. Davies, The epigenome and top-down causation, in *Interface focus*, 2012, Vol 2, pages 42-8, Online 14 September 2011,
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3262298/
(This overlaps with some of the ideas in the Meta-Morphogenesis project.)


Aaron Sloman, 2013, Meta-Morphogenesis and Toddler Theorems: Case Studies, Online discussion note, https://www.cs.bham.ac.uk/research/projects/cogaff/misc/toddler-theorems.html (also pdf). School of Computer Science, The University of Birmingham,


A. M. Turing, 1952, The Chemical Basis Of Morphogenesis, Phil. Trans. R. Soc. London B 237, 237, pp. 37--72,

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