Parallel Peaks: A Visualization Method for Benchmark Studies of Multimodal Optimization

Ran Cheng*, Miqing Li*, Xin Yao†*

*School of Computer Science, University of Birmingham, B15 2TT, Birmingham, UK
†Department of Computer Science and Engineering, Southern University of Science and Technology, 518055, Shenzhen, China

Email: ranchengcn@gmail.com, limitsing@gmail.com, xiny@sustc.edu.cn

Abstract—Multimodal optimization has attracted increasing interest recently. Despite the emergence of various multimodal optimization algorithms during the last decade, little work has been dedicated to the development of benchmark tools. In this paper, we propose a visualization method for benchmark studies of multimodal optimization, called parallel peaks. Inspired by parallel coordinates, the proposed parallel peaks method is capable of visualizing both distribution information and convergence information of a given candidate solution set inside a 2D coordinate plane. To the best of our knowledge, this is the first visualization method in the multimodal optimization area. Our empirical results demonstrate that the proposed parallel peaks method can be robustly used to visualize candidate solutions with a range of properties, including high-accuracy solutions, high-dimensional solution sets and solution sets with a large number of optima. Additionally, by visualizing the populations obtained during the optimization process, it can also be used to investigate search behaviors of multimodal optimization algorithms.

I. INTRODUCTION

Multimodal optimization (MMO) often refers to an optimization problem that involves multiple global optima. To solve an MMO problem, it is usually expected that an optimizer should obtain the multiple optimal solutions simultaneously. Population based metaheuristics such as evolutionary algorithms (EAs) provide a suitable solution framework, which maintains a set of candidate solutions during the optimization process. However, due to the poor capability of diversity preservation, most EAs originally designed for traditional single-objective optimization are not directly applicable to MMO. Therefore, to enhance the performance of traditional EAs in MMO, a variety of solution approaches have been proposed [1], [2], including niching approaches [3]–[7], modified variation operators [8]–[11], and the recently developed multiobjectivization approaches [12]–[16].

In spite of the various algorithms proposed for solving MMO problems, little work has been dedicated to the development of benchmark tools for MMO. Until recently, Li et al. have proposed a benchmark test suite for MMO in [17], known as the IEEE CEC’2013 benchmark test suite. In addition to the test functions, Li et al. have also suggested two performance measures: peak ratio (PR) and success rate (SR), where PR is used to count the number of global optima in a given solution set, and SR is used to check whether all global optima are found. Together with the test suite, PR and SR have been widely used for benchmark studies of MMO.

Since the research of MMO is still in its infancy [2], enriching benchmark tools is of particular interest. Apart from benchmark test functions and performance measures, visualization also plays an important role in performance investigation. Since there always exist multiple candidate solutions to be obtained in MMO, visualization becomes particularly useful in benchmark studies. On one hand, visualization can provide informative and intuitive measurements of a solution set by showing both distribution information and convergence information; on the other hand, visualization can help investigate the search behaviors of an algorithm via observations on the populations obtained during the optimization process.

Despite the importance of visualization as a benchmark tool, so far, no visualization method has yet been proposed in the MMO area. To fill this gap, this paper proposes a visualization method for benchmark studies of MMO, called parallel peaks, which can be used to visualize an arbitrary set of candidate solutions by mapping them on to 2D parallel coordinates [18]. In summary, the proposed parallel peaks method has the following characteristics:

- parameterless to be used for the visualization of candidate solutions;
- insensitive to the accuracy levels of candidate solutions;
- capable of indicating which global optima are found;
- capable of visualizing both distribution information and convergence information of candidate solutions in a 2D coordinate plane;
- scalable to the number of global optima;
- scalable to the number of candidate solutions;
- scalable to the dimension of candidate solutions.

The rest of this paper is organized as follows. Section II presents some background knowledge, including general definitions of MMO problems, formulations of performance mea-
expenses PR and SR, and basic concepts of parallel coordinates. Afterwards, Section III details the proposed parallel peaks method. To demonstrate the main properties of the proposed parallel peaks method and illustrate how it can be used in benchmark studies, Section IV presents some results of empirical assessments. Finally, Section V draws the conclusion.

II. BACKGROUND

As background knowledge, this section first presents the general definition of MMO problems and the formulations of PR and SR; then, a brief introduction to parallel coordinates is given; finally, the motivation of this work is articulated.

A. Multimodal Optimization

As shown in Fig. 1, an MMO problem often involves multiple global optima having the same fitness value, which can be generally formulated as follows1:

\[
\begin{align*}
\text{maximize } & f(x) \\
\text{s.t. } & x \in X
\end{align*}
\]

where \( f(x) \) is the objective function, \( x = (x_1, x_2, \ldots, x_D) \in X \) denotes a decision vector in decision space \( X \subseteq \mathbb{R}^D \). Given an MMO problem as formulated above, there exist a set of global optimal solutions \( X^* \) that maximize the objective function \( f(x) \) as:

\[
X^* = \{ x \in X : \forall y \in X : f(y) > f(x) \land y \neq x \},
\]

where \( |X^*| > 1 \) always holds.

\[\text{Without loss of generality, this paper only considers maximization problems.}\]

B. Performance Measures

To assess the quality of a solution set obtained in MMO, there are two criteria to be taken into consideration simultaneously: the number of global optima found by the algorithm and the convergence quality of each found optimum. To meet such a requirement, the peak ratio (PR) and the success rate (SR) are two widely adopted measures in the benchmark studies of MMO [17], [19]:

\[
PR = \frac{\sum_{run=1}^{NR} NPF_i}{NKP \times NR},
\]

and

\[
SR = \frac{NSR}{NR},
\]

where \( NR \) is the total number of trial runs, \( NPF_i \) is the number of global optima found in the \( i \)-th trial run, \( NKP \) and \( NSR \) are the number of known global optima and the number of successful runs, respectively. A trial run is known to be successful if and only if all global optima are found.

In the calculation of PR and SR, there are two parameters to be specified. First, in order to check whether a global optimum is found, a parameter \( \epsilon \) is used as the threshold accuracy level. It is believed that the global optimum is found if and only if the difference between the fitness value of a candidate solution and its closest global optimum is smaller than \( \epsilon \). Second, in order to tell whether two candidate solutions are different optima, a parameter \( r \) is also predefined for each test function as the threshold radius. To use PR and SR, one has to specify settings of \( \epsilon \) and \( r \) beforehand.

C. Parallel Coordinates

The approach of parallel coordinates is widely used in the visualization of high-dimensional datasets [18]. Essentially, values of the parallel coordinates can be seen as certain geometries in high dimensions transformed into easily observed 2D patterns.
Algorithm 1 Pseudo code of the proposed parallel peaks method.

1: **Input:** solution set to be visualized $S = (X, Y)$, positions of true global optima $X^*$, where $X = \{x_1, x_2, \ldots, x_n\}$, $Y = \{y_1, y_2, \ldots, y_n\}$, $X^* = \{x_1^*, x_2^*, \ldots, x_m^*\}$, and $n$ and $m$ are numbers of data points in $X$ and $X^*$ respectively;
2: **Output:** visualized figure Fig;
3: /*Data Transformation*/
4: $F = \emptyset$;
5: for $i = 1$ to $n$ do
6: $F_i = 1 - \frac{y_i}{y_{\text{max}}} - y_{\text{min}}$; // normalization of $y_i$
7: end for
8: $D = \emptyset$;
9: for $i = 1$ to $n$ do
10: for $j = 1$ to $m$ do
11: $D_{i,j} = ||x_i - x_j^*||$; // distance between $x_i$ and $x_j^*$
12: end for
13: end for
14: /*Visualization*/
15: $\text{Fig} \leftarrow$ empty parallel coordinates;
16: for $i = 1$ to $n$ do
17: $d_i = (D_{i,1}, D_{i,2}, \ldots, D_{i,m})$; // the $i$-the row of $D$
18: plot $d_i$ in $\text{Fig}$ using $F_i$ as grayscale color value;
19: end for

As illustrated in Fig. 2, all the axes in parallel coordinates are placed in parallel to each other, and each axis represents a dimension of the dataset to be visualized. In this way, given a dataset, regardless of the number of dimensions or data points, it can be always visualized inside a 2D plane using lines crossing the axes, where each line represents a data point.

**D. Motivation**

A desirable performance measure is expected to capture various features of a given solution set, especially for MMO where there exist multiple global optima having the same (or very similar) fitness value. Existing performance measures (such as PR and SR) which return a scalar value for a solution set fail to do so. It can be difficult for the user to comprehend the solution set by one or several scalar values.

To this end, this paper proposes a visualization method, called parallel peaks, aiming to allow solution sets obtained in the benchmark studies of MMO to be observed and assessed in an intuitive and comprehensive manner, such that the user can easily understand the search behaviors of an MMO algorithm.

**III. Method**

The basic idea of the proposed parallel peaks method is to map the distribution information and convergence information of a given solution set onto 2D parallel coordinates. In order to reflect the distribution information, the distance between each candidate solution and each global optimum is calculated and visualized. In addition, the fitness value of each candidate solution is reflected by a transformed grayscale color in the visualized figure.

As shown in Algorithm 1, the proposed parallel peaks method consists of two main components: data transformation component and visualization component. In the data transformation component, the input candidate solutions are transformed to the data to be visualized in parallel coordinates. First, as presented in Step 4 to Step 7, the fitness values are normalized into $[0, 1]$; afterwards, from Step 8 to Step 13, the distances between the input solutions and the true global optima are calculated.

With the transformed data, visualization is performed on a set of modified parallel coordinates, where $x$-axis represents the index of each true global optimum, and $y$-axis represents the distances between input candidate solutions and the global optima. To be specific, the distances (as stored in $D$) between the input candidate solutions and the true global optima are plotted in parallel coordinates using lines crossing the index of each global optimum, where the grayscale color of each line is calculated on the basis of the corresponding...
normalized fitness value (as stored in $F$). In this way, both distribution information and convergence information of the input candidate solutions are visualized inside the same 2D coordinate plane.

It is worth noting that, when using the normalized fitness values as grayscale information (Step 18 in Algorithm 1), different strategies may be applicable according to personal preferences. Here, we suggest using RGB vectors to represent the grayscale information as $[F_i, F_j, F_k]$, where $[0, 0, 0]$ and $[1, 1, 1]$ are colors of pure black and pure white respectively. In this way, the better a fitness value is, the darker the corresponding line will be in the figure. In case that a user is not interested in the fitness values of the candidate solutions, the RGB vector can be also constantly set to $[0, 0, 0]$.

To better understand the proposed parallel peaks method, we apply a recently proposed MMO algorithm [16] to the MMO test function shown in Fig. 1, and some typical populations obtained during the optimization process are recorded and visualized. As illustrated in Fig. 3, the proposed parallel peaks method is able to clearly reflect the distribution information of distances between candidate solutions and the true global optima. At the first generation, when the candidate solutions are randomly initialized, the lines in Fig. 3(b) are messy and indistinct, which means that most of the candidate solutions are distant from the true global optima and of poor fitness. After 50 generations, as shown in Fig. 3(c), the candidate solutions begin to converge towards each optimum, and correspondingly, the lines in Fig. 3(c) also become dense and darker. Finally, after the algorithm has run for 100 generations, a clear evidence of convergence can be observed from both Fig. 3(e) and Fig. 3(f). Further analyses on the main properties of the proposed parallel peaks method will be detailed in the following section.

IV. MAIN PROPERTIES

In order to demonstrate the main properties of the proposed parallel peaks method and illustrate how it can be used in benchmark studies, a series of experiments are conducted on the IEEE CEC’2013 benchmark test suite for multimodal optimization [17], where the main characteristics of the 20 test functions are summarized in Table I.

To be specific, the experiments to be conducted consists of two parts. First, we show how the proposed parallel peaks method can be used to visualize the solution sets obtained using two different state-of-the-art MMO algorithms on IEEE CEC’2013 benchmark test suite, namely, the multiobjective optimization for locating multiple optimal solutions of multimodal optimization problems (MOMMOP) [16] and the niching migratory multi-swarm optimizer (NMMSO) [20]. Then, we further demonstrate how the proposed parallel peaks method can be used to investigate the search behaviors of MMO algorithms such as MOMMOP and NMMSO.

The reason that we use MOMMOP and NMMSO to conduct the experiments is out of the following considerations. First, both MOMMOP and NMMSO are recently proposed state-of-the-art MMO algorithms, where NMMSO is the winner of IEEE CEC’2015 competition for multimodal optimization [21]. Second, the two algorithms perform very different search behaviors during the optimization process, where MOMMOP tries to find all optima simultaneously and NMMSO performs incremental search to locate each optimum successively. Therefore, using MOMMOP and NMMSO to conduct the experiments will provide some representative observations.

The experimental settings are as recommended in [17], where each algorithm is run for 50 independent times, and the termination condition for each test function is the maximum number of fitness evaluations (FEs) as summarized in Table II. The parameter settings of MOMMOP and NMMSO are as recommended in [16] and [20] respectively.

A. Visualization of Solution Sets

To evaluate the quality of solution sets obtained by MOMMOP and NMMSO, we still use the performance measures as introduced in Section II-B, namely, the peak ratio (PR) and success rate (SR). On the basis of the results listed in Table III and Table IV in Appendix A, we demonstrate how the proposed parallel peaks method can be used in different cases of benchmark studies.

1) Visualization of High-accuracy Solution Sets: As presented by Table III and Table IV in Appendix A, MOMMOP

<table>
<thead>
<tr>
<th>Function</th>
<th>No. of Global Optima</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1: Five-Uneven-Peak Trap (1D)</td>
<td>2</td>
</tr>
<tr>
<td>F2: Equal Maxima (1D)</td>
<td>5</td>
</tr>
<tr>
<td>F3: Uneven Decreasing Maxima (1D)</td>
<td>1</td>
</tr>
<tr>
<td>F4: Himmelblau (2D)</td>
<td>4</td>
</tr>
<tr>
<td>F5: Six-Hump Camel Back (2D)</td>
<td>2</td>
</tr>
<tr>
<td>F6: Shubert (2D)</td>
<td>18</td>
</tr>
<tr>
<td>F7: Vincent (2D)</td>
<td>36</td>
</tr>
<tr>
<td>F8: Shubert (3D)</td>
<td>81</td>
</tr>
<tr>
<td>F9: Vincent (3D)</td>
<td>216</td>
</tr>
<tr>
<td>F10: Modified Rastrigin</td>
<td>12</td>
</tr>
<tr>
<td>F11: Composition Function 1 (2D)</td>
<td>6</td>
</tr>
<tr>
<td>F12: Composition Function 2 (2D)</td>
<td>8</td>
</tr>
<tr>
<td>F13: Composition Function 3 (2D)</td>
<td>6</td>
</tr>
<tr>
<td>F14: Composition Function 3 (3D)</td>
<td>6</td>
</tr>
<tr>
<td>F15: Composition Function 4 (4D)</td>
<td>8</td>
</tr>
<tr>
<td>F16: Composition Function 3 (3D)</td>
<td>6</td>
</tr>
<tr>
<td>F17: Composition Function 4 (5D)</td>
<td>8</td>
</tr>
<tr>
<td>F18: Composition Function 3 (10D)</td>
<td>6</td>
</tr>
<tr>
<td>F19: Composition Function 4 (10D)</td>
<td>8</td>
</tr>
<tr>
<td>F20: Composition Function 4 (20D)</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test Functions</th>
<th>Maximum Number of FEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1 – F5</td>
<td>$5 \times 10^4$</td>
</tr>
<tr>
<td>F6, F7</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
<td>F8, F9</td>
<td>$4 \times 10^6$</td>
</tr>
<tr>
<td>F10 – F13</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
<td>F14 – F20</td>
<td>$4 \times 10^6$</td>
</tr>
</tbody>
</table>

All the solution sets to be visualized in the following are randomly selected from those obtained in the 50 trial runs.

TABLE I

<table>
<thead>
<tr>
<th>Test Functions</th>
<th>Maximum Number of FEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1 – F5</td>
<td>$5 \times 10^4$</td>
</tr>
<tr>
<td>F6, F7</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
<td>F8, F9</td>
<td>$4 \times 10^6$</td>
</tr>
<tr>
<td>F10 – F13</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
<td>F14 – F20</td>
<td>$4 \times 10^6$</td>
</tr>
</tbody>
</table>
and NMMSO show similar performance on most test functions, having achieved 100% PR and SR on F1, F2, F3, F4, F5, F7 and F10, where the performance of the two algorithms is not distinguishable at the accuracy level up to $\varepsilon = 10^{-5}$. In this case, a smaller accuracy level must be set appropriately in order to further distinguish the solution sets obtained by the two algorithms, which is, however, cumbersome in practice. To address such an issue, our proposed parallel peaks method provides an easy and intuitive solution approach.

As shown in Fig. 4, the solution sets obtained by MOM-MOP and NMMSO on F4 and F10 are visualized using the proposed method. Although the performance of MOMMOP and NMMSO cannot be distinguished by PR and SR on these two functions, their visualized solutions sets are significantly different. First, the candidate solutions in the solution set obtained by MOMMOP are generally much closer to each optimum than those obtained by NMMSO, especially on F10. Second, the solution set obtained by MOMMOP has a higher solution density around each optimum than NMMSO. On the basis of the above observations, it can be concluded that the proposed parallel peaks method can be used to distinguish solution sets at high accuracy levels.

2) Visualization of Low-accuracy Solution Sets: While MOMMOP and NMMSO have achieved generally promising performance on F1 to F10, their performance deteriorates rapidly on composite functions from F11 to F20. Especially on F16 to F20 which are of high dimensions, both algorithms fail to achieve any successful run, thus resulting in $SR = 0$ at all accuracy levels as presented by Table IV in Appendix A. In this case, the proposed parallel peaks method still can be used to investigate the quality of the solution sets, regardless of the accuracy levels.

As exemplified in Fig. 5, although the candidate solutions obtained by MOMMOP and NMMSO on F18 and F19 are of low accuracies, the proposed parallel peaks method is still able to present them inside 2D parallel coordinates. As can be observed, due to particular complexity of the search space, MOMMOP and NMMSO have only managed to find part of the global optima on both functions. Another interesting observation is that the candidate solutions found by MOMMOP and NMMSO on F19 belong to three different optima, which implies different search behaviors of the two algorithms. In Section IV-B, we will further demonstrate how to investigate the search behaviors of MOMMOP and NMMSO using the proposed parallel peaks method.

3) Visualization of Solution Sets with a Large Number of Optima: As summarized in Table I, although most multimodal landscapes only contain a small number of global optima (e.g. 6 or 8), in some extreme cases, there may exist a large number of global optima to a highly multimodal problem such as F8 or F9. In order to demonstrate the method’s scalability to the number of optima, solution sets obtained by MOMMOP and NMMSO on F8 are visualized in Fig. 6.

Consequently, in spite of the large number of global optima, the proposed parallel peaks method still works well, indicating two important observations. First, as evidenced by Fig. 6(a), MOMMOP has a stable performance on F8 as each obtained candidate solution has a similar distance to the closest optimum. Second, as evidenced by Fig. 6(b), NMMSO has obtained a large number of candidate solutions after one run, some of which are very close to the global optima while the others are relatively distant. However, indicated by the light colors of most lines in the figure, many candidate solutions obtained by NMMSO are just local optima which have poor
B. Investigation of Search Behaviors

In this subsection, we empirically demonstrate how to investigate the search behaviors of MMO algorithms using the proposed parallel peaks method. To exemplify, we apply Mommop and NMMSO to F11 whose fitness landscape can be referred to Fig. 7.

Some representative populations obtained by Mommop during the optimization process are visualized in Fig. 8, where there are three important observations. First, regardless of the search stage, the distances between the global optima and the nearby candidate solutions are always very similar. This observation indicates that the population of Mommop performs parallel convergence towards each optimum simultaneously. Second, as indicated by the light-colored lines in Fig. 8(c), the candidate solutions around optimum 3 and optimum 4 have relatively worse fitness than those around the other four optima. This observation implies that optimum 3 and optimum 4 distribute on sharp peaks in the fitness landscape, which is consistent with the observation in Fig. 7.

For NMMSO, some representative populations are visualized in Fig. 9, where it turns out that NMMSO has completely different search behaviors in contrast with Mommop. To be specific, there are two important observations. First, the number of located optima increases as the optimization process proceeds, which indicates that the population of NMMSO performs sequential convergence to each optimum successive-
ly. Second, as evidenced by Fig. 9(c) and Fig. 9(d), once an optimum is located, NMMSO is able to maintain the corresponding candidate solution in the population and to keep improving its convergence quality. This happens even if the solutions are distributed on a very sharp peak such as that indexed by 3 or 4 in Fig. 7.

V. Conclusion

The infancy of the MMO research requires new input not only on algorithm designs but also on test functions and performance measures. A desirable performance measure is expected to capture various behavior of a given solution set, especially for MMO where there exist multiple global optima having the same (or very similar) fitness value. Existing performance measures which return a scalar value for a solution set may fail to do so. It can be difficult for the user to comprehend the solution set by one or several scalar values. To this end, this paper has proposed a visualization method, called parallel peaks, to help assess and investigate solution sets obtained in benchmark MMO. Inspired by parallel coordinates, the proposed parallel peaks method is capable of visualizing both distribution information and convergence information inside a 2D coordinate plane, where the distribution information is reflected by the distances between candidate solutions and true global optima, and the convergence information by the grayscale colors transformed from fitness values.

Compared to performance measures which only return a scalar value for a solution set, the proposed parallel peaks method is able to visualize every candidate solution in parallel coordinates, thus providing comprehensive assessment and investigation of the solution set. More importantly, it can be used to measure candidate solutions of arbitrary accuracy levels without any parameter setting.

To demonstrate the main properties of the proposed parallel peaks method and how it can be used, a series of empirical analyses have been conducted on a variety of solutions sets obtained in different cases of benchmark studies, including high-accuracy solution sets, high-dimensional solutions sets, and solution sets with a large number of optima. In addition, we have also demonstrated that the proposed parallel peaks method can also be used to visualize populations obtained during the optimization process, which is particularly meaningful in the investigation of search behaviors of MMO algorithms.

We hope that the proposed parallel peaks method can promote the future development of MMO and call for more research dedicated to this field. In subsequent work, we would like to investigate whether it is possible to extend this method to reflect some other useful information, e.g., gradient information of the fitness landscapes.

ACKNOWLEDGMENT

The authors would like to thank Dr. Michael Epitropakis and Dr. Xiaodong Li for providing the code for generating true global optima of test functions in the IEEE CEC’2013 benchmark test suite. This work was supported by grants from EPSRC, Projects EP/K001523/1 and EP/J017515/1.

REFERENCES

APPENDIX A
RESULTS OF BENCHMARK EXPERIMENTS CONDUCTED IN SECTION IV

TABLE III
THE MEAN PEAK RATIOS (PRs) AVERAGED OVER 50 RUNS AT DIFFERENT ACCURACY LEVELS. BEST RESULTS ARE HIGHLIGHTED.

<table>
<thead>
<tr>
<th>ε</th>
<th>MOMMOP</th>
<th>NMMSO</th>
<th>MOMMOP</th>
<th>NMMSO</th>
<th>MOMMOP</th>
<th>NMMSO</th>
<th>MOMMOP</th>
<th>NMMSO</th>
<th>MOMMOP</th>
<th>NMMSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^-1</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^-2</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^-3</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^-4</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^-5</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE IV
THE MEAN SUCCESS RATES (SRs) AVERAGED OVER 50 RUNS AT DIFFERENT ACCURACY LEVELS. BEST RESULTS ARE HIGHLIGHTED.

<table>
<thead>
<tr>
<th>ε</th>
<th>MOMMOP</th>
<th>NMMSO</th>
<th>MOMMOP</th>
<th>NMMSO</th>
<th>MOMMOP</th>
<th>NMMSO</th>
<th>MOMMOP</th>
<th>NMMSO</th>
<th>MOMMOP</th>
<th>NMMSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^-1</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^-2</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^-3</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^-4</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10^-5</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>