Diffuse Optical Tomography based on Simplified Spherical Harmonics Approximation

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Abstract: The use of higher order approximations to the Radiative transport equation, through simplified spherical harmonics expansion (SP\textsubscript{N}) in optical tomography are presented. At higher orders, the image reconstruction becomes highly under-determined due to the large increase in the number of unknowns which cannot be adequately recovered. However, reconstruction of diffuse parameters, namely optical absorption and reduced scatter have shown to be more accurate where only the sensitivity matrix used in the inverse problem is based on SP\textsubscript{N} method and image reconstruction is limited to these two diffuse parameters.

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1. Introduction

The use of biological markers for function specific optical imaging has led to the development of novel imaging systems and algorithms that aim to recover images of function specific activity within small animals using, for example, fluorescence [1] or bioluminescence markers [2, 3]. Most NIR imaging systems rely on the use of a model based image reconstruction algorithm in which the light distribution within the imaging domain is approximated to provide a match to measured boundary data. The most commonly applied model for the estimation of NIR light propagation in tissue is achieved through the Diffusion Approximation (DA) to the Radiative Transport Equation (RTE) through its robustness in implementation and computational speed and flexibility. To accurately model NIR light propagation in small geometries, or models with high absorption (such as for bioluminescence imaging), higher ordered forward models are needed whereby the problem is no longer limited by the ‘diffuse’ approximation. The application of Simplified Spherical Harmonics (SP\textsubscript{N}) approximation to the RTE has been previously applied and studied [4, 5] and have been demonstrated to give accurate solutions for small geometries and in cases where the source/detector separation is small.

The vast majority of existing model based reconstruction algorithms make use of the DA [6, 7]. The image reconstructions therefore aim to form images of the absorption coefficient, \(\mu_a\) and the diffusion coefficient \(\kappa = 1/3(\mu_a + \mu_s')\) where \(\mu_s' = (1-g)\mu_s\) and \(\mu_a\) and g are the scattering coefficient and anisotropic factor respectively. There are been limited work on the feasibility of using higher order SP\textsubscript{N} methods in image reconstruction, whereby the number of unknowns increase for each increased order of the approximation. In this study, SP\textsubscript{N} based models are used to present a framework for image reconstruction in NIR imaging, whereby the parameters to be reconstructed are limited to absorption and reduced scattering coefficient.

2. Methods and Results

SP\textsubscript{N} based image reconstruction algorithms have been developed for \(N=1, 3, 5\) and 7 using the modified Tikhonov minimization method. If we assume that the composite moments of fluence are measurable, the SP\textsubscript{N} approximation introduces the possibility of reconstructing a wider range of optical parameters, Table 1. Test boundary data was generated using a 2D circular geometry of radius 43 mm, with \(\mu_a = 0.01\) mm\(^{-1}\), \(\mu_s = 10\) mm\(^{-1}\), and g = 0.9. A highly absorbing anomaly, with \(\mu_a = 0.02\) mm\(^{-1}\) and a highly scattering anomaly with \(\mu_s = 20\) mm\(^{-1}\), were inserted, Figure 1.

Each of the SP\textsubscript{N} reconstruction algorithms (for \(N = 1, 3, 5\) and 7) is capable of reconstructing different optical properties, depending on the order N. In order to test the capability of each of the SP\textsubscript{N} models, the optical properties listed in Table 1 were reconstructed, and from these, values of \(\mu_a\) and \(\mu_s\) were extracted [5]. For the inverse model, the same mesh as the forward model was used without added noise to ensure that any errors in the reconstructed optical maps were only due to the complexity of the higher ordered approximations.

Figure 2(a) shows the reconstructed images generated using an SP\textsubscript{1} based reconstruction algorithm and SP\textsubscript{1} forward data. The location of the two anomalies has been accurately reconstructed. The background optical
properties have been accurately recovered although the values of the two anomalies have been over-estimated. The reconstructed image shows a maximum absorption coefficient of 0.0249 mm\(^{-1}\) and a maximum scattering coefficient of 23.9 mm\(^{-1}\). The SP\(_1\) reconstruction required 32 iterations at 3.1 second per iteration to recover the two relevant unknowns.

<table>
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<th>Table 1. Reconstructed values of SP(_N) algorithms where the diffusion terms (k_n = 1/(A\mu_n)) and A is a constant.</th>
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<td><strong>Reconstruction model</strong></td>
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<td>SP(_1)</td>
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<td>SP(_5)</td>
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<td>SP(_7)</td>
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The SP\(_3\) reconstruction (using SP\(_3\) data) Figure 2(b) has also accurately recovered the location and shape of the two anomalies. The recovered absorption and scattering coefficients have underestimated the target values, returning 0.0189mm\(^{-1}\) and 17.3mm\(^{-1}\) respectively. Unlike the SP\(_1\) image, the SP\(_3\) reconstruction shows signs of cross talk between the absorption and scattering images. The SP\(_3\) reconstruction required 13 iterations at 7.3 second per iteration to recover the four relevant unknowns.

Reconstruction using the SP\(_5\) model (using SP\(_5\) data) is shown in Figure 2(c). In this case, the optical properties have again been overestimated with a maximum absorption coefficient of 0.0224mm\(^{-1}\) and a maximum scattering coefficient of 21.4mm\(^{-1}\). The cross talk between the absorption and scattering images is still present. The SP\(_5\) reconstruction required 34 iterations at 54 second per iteration to recover the six relevant unknowns.

The optical maps generated by the SP\(_7\) model (using SP\(_7\) data), Figure 2(d) contains artifact throughout the domain. The location of the highly absorbing target has been recovered with poor size and contrast accuracy. The reconstruction failed to recover the highly scattering target. It is likely that the failure of the SP\(_7\) reconstruction is due to the highly under-determined nature of the problem. The SP\(_7\) model requires 8 unique variables to be calculated at each node, i.e. 14280 unknowns, based on just 1920 boundary measurements. The SP\(_7\) reconstruction required 20 iterations at 52 second per iteration to recover the eight relevant unknowns.

The SP\(_N\) approximations are based on composite moments of fluence with the total flux calculated from weighted sum of these components [5]. The SP\(_N\) reconstruction algorithms used above were therefore based on various moments of the phase and amplitude boundary data which attempted to recover all moments of absorption and scattering coefficients. In practice, however, there are no experimental methods to easily measure the angular components of amplitude and phase and so image reconstructions must be based on total values of fluence. A new reconstruction algorithm was therefore developed that used the SP\(_N\) model but only required measurements of phase and amplitude as available from the total fluence and not composite moments. The new reconstruction algorithm uses the SP\(_5\) forward model to calculate the composite moments of fluence. The Jacobian is then constructed where log of amplitude and absolute phase are extracted from the calculated total fluence from the SP\(_N\) model. By eliminating the composite moments of fluence, however, it is only possible to reconstruct for \(\mu_a\) and \(\mu_s\) (assuming \(g = 0.9\)). This new reconstruction algorithm is tested using SP\(_5\) forward data and the resulting optical map is shown in Figure 3(a). The reconstruction has performed well, recovering the optical parameters within 10% of the target.
values. For comparison, the same forward data is also used to reconstruct optical parameters using the DA based algorithm, Figure 3(b). In this case, the target values have been over-estimated and boundary artifact has been introduced, indicating the maximum errors seen from high order model mismatch, as is expected near the source / detector positions, which lead to image inaccuracy and artifacts.

3. Discussions and conclusions
Reconstruction algorithms based on the SPN approximation have been developed and tested. For image reconstruction, the same FEM model as for the forward data was used and no noise was added. It is shown, Figure 2(a)-(c), that for N=1, 3 and 5 the reconstructions performed well, with the reconstructed values being within 24% of expected values with the worse results being obtained from SP1 and absorption coefficient. This accuracy could be further improved by the optimization of the regularization parameter and stopping criteria. The N=7 model, however, failed to accurately recover the target values, Figure 2(d). The absorption coefficient, although located, is extremely over estimated, with some cross-talk from the scattering value. The reconstructed scatter image contains high valued artifacts and can be considered inaccurate. By limiting the presented results to these larger domains, it is also worth noting that the majority of image artifacts are seen at the boundary, near the source locations, where SP1 solution is known to be less accurate. It is therefore expected that the errors seen due to the lack of higher order approximations to be substantially more significant in small geometry imaging experiments. The poor performance of the SP7 model was most likely due to the under-determined nature of the problem. As stated earlier, the SP7 model contains 8 unique unknown variables which need to be calculated at each spatial location (FEM node), i.e. 14280 unknowns, based on just 1920 boundary measurements (240 log amplitude and 240 Phase, based on 16 co-located source and detectors and 4 composite moments of fluence).

The SPN equations are based on composite moments of fluence [5]. In reality, however, experimental systems can only measure the total fluence at the boundary of the domain and so the full SPN reconstruction algorithms are of limited use in image reconstruction where all composite moments are needed. An alternative method has been proposed in which the forward models are based on absolute fluence but the Jacobian matrix was calculated using the SP3 model to allow the calculation of absorption (μa) and scattering (μs) coefficient only. The diffusion parameter based images reconstructed from simulated SP2 data whereby the Jacobian is based on either SP1 or SP3, Figure 3, indicate that although both models can be used, the higher order model is more accurate both in terms of quantitative and qualitative analysis. The target values of the test problem calculated using SP2 are recovered with more accuracy as compared to the SP1 based model with much less artifact. Additionally, eliminating the use of complex moments also results in decreased computation time and memory requirements.

The results presented in this work indicate that for image reconstruction whereby the DA is less valid, in for example, small animal imaging and / or where the absorption coefficient is more dominant, the higher order models based on simplified spherical harmonics can be used to generate the sensitivity matrix for diffusion based image reconstruction, without the additional computation complexity in terms of the number of unknown parameters. The incorporation of these more accurate models can however allow for a better accuracy in terms of light propagation models.

4. Acknowledgements
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5. References