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The pain and gain of DC-based diffuse optical tomography reconstruction---New insights into an old-like problem

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Abstract: For diffuse optical tomography reconstruction, DC-based method outperforms frequency-domain method in background artifacts, at the known cost of increased coupling between absorption and scattering. The differences of these methods diminish when spatial *priors* are available.

OCIS codes: (170.3880) Medical and biological imaging; (170.3010) Image reconstruction techniques; (170.6960) Tomography; (170.5270) Photon density waves

1. Introduction

Diffuse optical tomography quantifies the spatial heterogeneities of NIR absorbing chromophors and scattering particles by measurement of light diffused through biological tissue. Steady-state and frequency-domain (FD) measurements are most commonly utilized to reconstruct the tissue absorbance and scattering distributions. Steady-state system only measures the attenuation of the direct-current (DC) amplitude of the photon while frequency domain system ideally acquires the same DC, the amplitude of the modulated light intensity (AC), and the phase of the modulation of the light intensity (referred to as “Phs” in this paper). The role of DC component in FD reconstruction has yet to be comprehensively analyzed and the confidence level of reconstruction with only the DC information available has yet to be clearly understood.

This paper compares the image reconstructions using three sets of measurements, which are DC only, AC/Phs and DC/AC/Phs, to evaluate the role of DC information in diffuse optical tomography reconstruction. It is found that, DC-based method outperforms FD method in background artifacts, at the known cost of increased coupling between absorption and scattering, and the differences of the methods diminish when spatial *prior* can be implemented.

2 Theory

Under the assumption of accurate forward computational model to describe the light propagation, it is necessary to consider two factors when evaluating the overall performance of the reconstruction: First, the assembled measurement error that could be mapped to the uncertainty in image reconstruction; Second, the determinacy of the inverse problem.

2.1 Analyses of the parameter recovery uncertainty caused by assembled measurement error (PRUAME)

The measurements for both FD and CW systems are typically governed by the diffusion approximation to the radiative transfer equation[1]. For the simplest case of recovering the optical properties of an infinite homogeneous medium, the photon densities for DC and FD measured at a position $\vec{r}\,'$ that has a distance of d from a source at $\vec{r}\,'$ are:

$$U_{DC}(\vec{r},0)=\frac{S_{DC}(\vec{r}\,',0)}{4\pi vDd}\exp(-\sqrt{\frac{\mu_a}{D}}d) \tag{1}$$

$$U_{AC}(\vec{r},\omega)=\frac{S_{AC}(\vec{r}\,',\omega)}{4\pi vDd}\exp(-d\sqrt{\frac{\mu_a}{2D}\left(\sqrt{1+\frac{\omega^2}{v^2{\mu_a}^2}}+1\right)})\cdot\exp(id\sqrt{\frac{\mu_a}{2D}\left(\sqrt{1+\frac{\omega^2}{v^2{\mu_a}^2}}-1\right)}) \tag{2}$$

Therefore the measurements made at source-detector separations of d_1 and $d_2=d_1+\rho$, respectively, may results in the following parameters: δ --attenuation of steady state light intensity (DC); α --attenuation of the amplitude of the modulated light intensity (AC); ϕ --phase shift of the modulation of the light intensity (Phs), as

$$\delta=\ln\frac{(d_2|U_{DC}(d_2)|)}{(d_1|U_{DC}(d_1)|)}=-\rho\cdot\sqrt{\frac{\mu_a}{D}};\ \alpha=\ln\frac{(d_2|U_{AC}(d_2)|)}{(d_1|U_{AC}(d_1)|)}=-\rho\cdot\sqrt{\frac{\mu_a}{2D}\left(\sqrt{1+\frac{\omega^2}{v^2{\mu_a}^2}}+1\right)};\ \phi=\Phi(d_2)-\Phi(d_1)=\rho\cdot\sqrt{\frac{\mu_a}{2D}\left(\sqrt{1+\frac{\omega^2}{v^2{\mu_a}^2}}-1\right)} \tag{3}$$

For small variations of the source-detector distance among different source-detector pairs, the signal variations may actually be sensed as the "assembled measurement error" [2]. Suggested by [2], the "**parameter recovery uncertainty caused by assembled measurement error"** (PRUAME) is derived for the reconstruction methods of DC, AC/PHs, and DC/AC/Phs, with respect to each unknown quantities, as shown in Table 1.

Table 1 PRUAME expressions.

	$\sigma_{\mu a}/\mu_a$		σ_D/D	$\sigma_{\mu s'}/\mu_s'$
DC	$2\cdot\left(\frac{\sigma_\delta^2}{\delta^2}\right)^{1/2}$ [2]		$2\cdot\left(\frac{\sigma_\delta^2}{\delta^2}\right)^{1/2}$	$\left[\left(\frac{1}{3D}\right)^2+\mu_a^2\right]^{1/2}\cdot 2\left(\frac{\sigma_\delta^2}{\delta^2}\right)^{1/2}\cdot\left[\frac{1}{3D}-\mu_a\right]^{-1}$
	Est. Val.	2	2	2
AC/ Phs	$\frac{\alpha^2+\phi^2}{\alpha^2-\phi^2}\left(\frac{\sigma_\phi^2}{\phi^2}+\frac{\sigma_\alpha^2}{\alpha^2}\right)^{1/2}$ [2]		$\left(\frac{\sigma_\alpha^2}{\alpha^2}+\frac{\sigma_\phi^2}{\phi^2}\right)^{1/2}$	$\left[\left(\frac{1}{3D}\right)^2\left(\frac{\sigma_\alpha^2}{\alpha^2}+\frac{\sigma_\phi^2}{\phi^2}\right)\cdot+\mu_a^2\cdot\left(\frac{\alpha^2+\phi^2}{\alpha^2-\phi^2}\right)^2\cdot\left(\frac{\sigma_\alpha^2}{\alpha^2}+\frac{\sigma_\phi^2}{\phi^2}\right)\right]^{1/2}\cdot\left[\frac{1}{3D}-\mu_a\right]^{-1}$
	Est. Val.	1.6189	1.4142	~1.4142
AC/ DC/ Phs	$\left(\frac{\sigma_\phi^2}{\phi^2}+\frac{\sigma_\alpha^2}{\alpha^2}+4\frac{\sigma_\delta^2}{\delta^2}\right)^{1/2}$		$\left(\frac{\sigma_\alpha^2}{\alpha^2}+\frac{\sigma_\phi^2}{\phi^2}\right)^{1/2}$	$\left[\left(\frac{1}{3D}\right)^2\left(\frac{\sigma_\alpha^2}{\alpha^2}+\frac{\sigma_\phi^2}{\phi^2}\right)\cdot+\mu_a^2\cdot\left(4\frac{\sigma_\delta^2}{\delta^2}+\frac{\sigma_\alpha^2}{\alpha^2}+\frac{\sigma_\phi^2}{\phi^2}\right)\right]^{1/2}\cdot\left[\frac{1}{3D}-\mu_a\right]^{-1}$
	Est. Val.	2.4495	1.4142	~1.4142

To quantitatively compare the magnitude of the expressions, optical properties close to those of actual tissue, μ_a=0.005mm^{-1}, μ_s'=1mm^{-1}, and source-detector separation of ρ=10mm are substituted into the previous equations. Further assumptions are made by the assumption that the error magnitudes are the same for all the measurements ($\frac{\sigma_\delta^2}{\delta^2}\cong\frac{\sigma_\alpha^2}{\alpha^2}\cong\frac{\sigma_\phi^2}{\phi^2}$) as indicated in [2]. By normalizing the values along column 2 and 3 with $\frac{\sigma_\delta}{\delta}$ and column 4 with $\frac{\sigma_\delta}{\delta}\cdot\left[\frac{1}{3D}-\mu_a\right]^{-1}$, the reconstruction uncertainties are given in Table 1 as the "estimated value".

Comparison in Table 1 indicates that from only the PRUAME perspective, AC/Phs possesses the least overall reconstruction uncertainty, followed by AC/DC/Phs and DC only.

With ref [3,4], the above analyses for the PRUAME comparisons based on infinite medium can be extended to semi-infinite medium and reaches qualitatively similar estimations.

2.2 Inverse problem determinacy

The inverse problem includes two scenarios. When the spatial *prior* is unavailable, more independent measurements are desired to reduce the under-determinacy condition of piecewise reconstruction. Under such consideration, DC/AC/Phs measurement could be the most deterministic measurement combination, although DC components are sometimes ignored in the sense that it may be redundant to the AC components. However, by comparing the 2nd and 3rd sub-equations in equ. (3), it can be concluded that the AC attenuation usually is not linearly proportional to the DC attenuation. Therefore DC information may be necessary for complete recovery of tissue properties.

When a complementary imaging modality is available to provide hard *a prior* to the image reconstruction [5], the inverse problem becomes over-determined. Under such condition, it is imperative to know how well DC-based image reconstruction performs as compared to the cases of having FD information available.

3 Simulations

Numerical simulations are conducted to investigate the validity of the above theoretical analyses. The forward model is formulated with finite element method based on diffusion approximation and Robin type boundary condition [6]. The sensitivity matrices (Jacobian) are structured as the one in below, for each measurement category in Fig. 1&2

$$J=\left[\frac{\partial\ln I_{AC}}{\partial\mu_a}\quad\frac{\partial\ln I_{AC}}{\partial D};\quad\frac{\partial\phi}{\partial\mu_a}\quad\frac{\partial\phi}{\partial D};\quad\frac{\partial\ln I_{DC}}{\partial\mu_a}\quad\frac{\partial\ln I_{DC}}{\partial D}\right]\qquad(4)$$

The DC/AC/Phs combination utilizes all the measurements so it contains all terms shown in equ. (4); while for CW method, only the last two terms in equ (4) are used and the first four terms are retained for AC/Phs method. The Levenberg-Marquardt algorithm is integrated as the inverse solver for the simulative evaluations.

3.1 Piece-wise simulation

The simulation is to solve for the optical properties at 2760 nodes in FEM mesh 240 (16×15), the location and maximum optical properties (shown on the bar chart) within each target region are shown in Fig.1. For the target profile and optical property recovery, DC only reconstruction demonstrates lowest accuracy and most significant crosstalk. DC/AC/Phs outperforms AC/Phs in most cases, especially for the μ_a contour of target 3 and μ_s'/D value recovery of target 2. However, the background variations (σ^2 value of each reconstructed image) indicate that DC only reconstruction presents the best background homogeneity, followed by DC/AC/Phs and AC/Phs. The background homogenity in image reconstruction is espacially important for DOT of prostate cancer, because the cancer target is to be resolved within the optically heterogeous prostatic tissue.

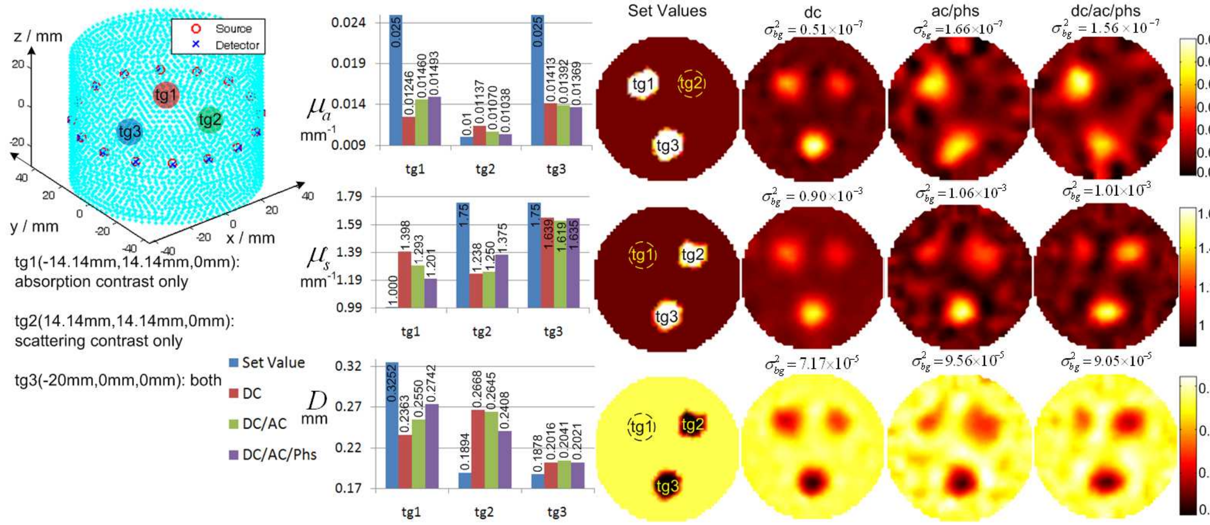

Fig.1 Piece-wise reconstruction

3.2 Region-wise simulation

With the same setup as the piecewise simulation and the assumption that the target region can be accurately segmented, region-wise reconstructions found that the DC only method, having less measurements, performs equivalently to the two methods with the FD information included.

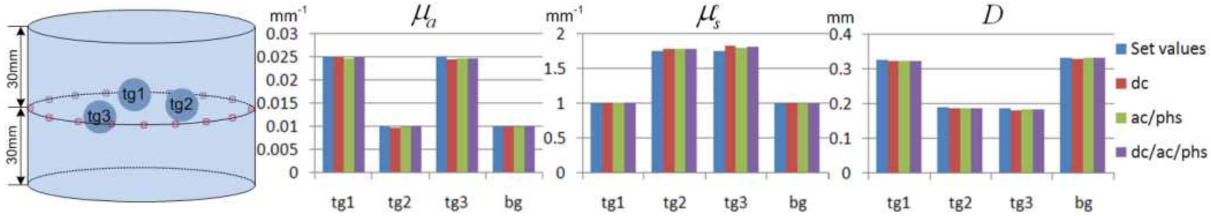

Fig.2 Region-wise reconstruction

4 Conclusions

The theoretical analysis and numerical studies have several implications to make: (1) DC-only piece-wise reconstruction outperforms other methods in background artifacts reduction but its performance on the target recovery accuracy and cross coupling suppression is less desirable; (2) DC/AC/Phs approach shows superiority over AC/Phs in piecewise reconstruction; (3) DC only region-wise reconstruction is equivalent to that based on FD system when the spatial *a priori* constraint is available.

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