Concrete Data Structures as Dialogue Games

Clément Jacq  Paul-André Melliès

Institut de Recherche en Informatique Fondamentale
CNRS & Sorbonne Paris Cité

Dialogue games were introduced by Mellies [4,5] as a means to unify the two traditional but still largely disconnected formulations of game semantics:

- by the concrete data structure interpretation of PCF designed by Berry and Curien in the early 1980s [1],
- by the arena game model of PCF designed by Hyland, Nickau and Ong in the mid 1990s [2,6].

The guiding idea underlying the notion of dialogue game is that every move \( m \) of an arena game should be decomposed as a pair \( m = (\alpha, val) \) consisting of a cell \( \alpha \) and of a value \( val \). In that way, every move \( m \) of the arena game becomes the action for Player or for Opponent of filling a given cell \( \alpha \) with a given value \( val \). The translation is achieved thanks to an important novelty of dialogue games with respect to concrete data structures: the fact that a cell \( \alpha \) as well as a value \( val \) may be either Opponent or Player. The only pair of constraints is that (1) a value should always fill a cell of the same polarity, and (2) a value should always justify a cell of the opposite polarity. Once accepted this decomposition of an arena move as a pair \( m = (\alpha, val) \) consisting of a cell and of a value, the connection between dialogue games and arena games becomes essentially immediate. In contrast, and despite the fact that they share the same philosophy of cells and values, the connection between dialogue games and concrete data structures is more subtle.

In this talk, we will explain how to connect the sequential algorithm model designed by Berry and Curien in the early 1980s [1] with the more recent notion of dialogue game [4, 5]. To that purpose, we start from the graph game model of intuitionistic linear logic designed by Hyland and Schalk [3] in the early 2000s. We establish a number of interesting properties of concrete data structures by embedding graph games into dialogue games. One benefit of this embedding is that it enables us to apply to graph games the graphical notations of positions in dialogue games. Typically, the formula \((\text{Bool} \otimes \text{Bool}) \to \text{Bool}\) is interpreted as the graph game \(A\) and at the same time as the dialogue game \(B\) associated to the formula of tensorial logic:

\[
B = \neg \left( \neg \neg (1 \oplus 1) \otimes \neg \neg (1 \oplus 1) \otimes \neg (1 \oplus 1) \right)
\]

Seen in that way, every node of the graph game \(A\) becomes a specific position of the dialogue game \(B\), and may be thus depicted in the following way, see [4,5] for details:

Besides the connection with dialogue games, the main technical contribution of the paper is a reformulation of the notion of sequential algorithm by Berry and Curien in the total case, adapting to dialogue games the existing description by Hyland and Schalk in graph games.
References


